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MATHEMATICAL EDUCATION IN THE OTTOMAN EDUCATIONAL SYSTEM: THE CASE OF *CĀMÎ'Ü'L-ĤİSĀB*

Osmanlı Eğitim Sisteminde Matematik Eğitimi: Cāmi'ü'l- Ĥisāb Örneği

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Abstract: The linguistic aspects as well as stylistic features of course books matter as much as their contents and place in the curriculum. In the Ottoman lands, scientific education was made in Arabic and the course books were prepared accordingly. This was as well the case for mathematics. However, some manuals in this field were prepared in Turkish, the foremost vernacular language. Turkish must have been used especially as a medium for the initiation to the subject matter while its use must have been related to students' linguistic capacities in Arabic and/or Persian. In this article, *Cāmi'ü'l-Ĥisāb*, written by Yūsuf bin Kemāl el-Bursevî, a former student of İskender Çelebi, a renowned *defterdar* of Suleiman the Magnificent which was completed in Muharrem 934 (October/November 1527) is analyzed. Our focus here is the examination of the stylistic features of the text as well as the use of Turkish as a pedagogic medium.

Keywords: *Cāmi'ü'l-Ĥisāb*, Ottoman Scientific Curriculum, Maths, Pedagogical Methods, Pedagogical Style

Öz: Osmanlı medreselerinde okutulan ders kitaplarının içerik, okutulma sıklıkları kadar dili de önemlidir. Osmanlı'da bilim dilinin Arapça olması sebebiyle işlenen ders kitaplarının çoğunluğunun dili Arapçadır. Bu durum matematik ders kitapları için de geçerlidir. Arapça eserlerin yanında Türkçe yazılmış matematik ders kitapları da vardır. Eğitimde Türkçe, daha çok öğrenciye ulaşmak için bir aracı dil konumundadır. Bu sebeple, öğrencinin bildiği dil üzerinden bir anlatım gerçekleşir. Yazarın kullandığı dil, üslup ve matematik terimlerinin kullanımı muhatapın sadece Türkçe veya Türkçe yanında Arapça ve / veya Farsça bilmesine göre değişiklik gösterir. Bu çalışmada Kanunî dönemi baş defterdarı İskender Çelebi'den dersler almış olan Yūsuf bin Kemāl el-Bursevî tarafından Muharrem 934'te (Ekim / Kasım 1527) yazılmış *Cāmi'ü'l-Ĥisāb* adlı eserden hareketle matematiğin öğretim dili muhatap, yazar / öğretici arasındaki ilişki dikkate alınmış ve Türkçenin yapısı üzerinden değerlendirilmiştir. Buradaki odağımız, metnin üslup özelliklerinin incelenmesi ve Türkçenin pedagojik bir ortam olarak kullanılmasıdır.

Anahtar Kelimeler: *Cāmi'ü'l-Ĥisāb*, Osmanlı Bilimsel Ders Programı, Matematik, Pedagojik Yöntemler, Pedagojik Üslup

Introduction

Mathematics was, as it is the case in the Western Scholastic system, a part of the *Quadrivium* in the Middle Eastern educational institutions, comprising arithmetic, geometry, astronomy and music.¹ Ottoman madrasa course books were prepared, with some exceptions, in Arabic and mathematical works, either *muhtasar* (summary) or *müfîd*, were also in that language. Among these, treatises written by Nizamüddin el-Nisâburî, İbnü'l-Havvâm Kemâlüddin el-Fârisî, İbnü'l-Hâim, Bahâuddin el-Âmili and Ali Kuşçu were largely read and taught.² There were, however, depending on pedagogical needs, some manuals prepared in

¹ C. İzgi, *Osmanlı Medreselerinde İlim*, C.I Riyaziyyat, İz Yayıncılık, İstanbul, 1997, p. 208. When compared to the astronomical literature, arithmetical works were Turkified in a later period and when compared to the geometrical one, in an earlier phase.

² Cevat İzgi, *op. cit.*, p. 207-252.

Turkish, such as Hacı Atmaca's *Mecmâ'ü'l-Kavâ'id* and Matrakçı Nasuh's *Umdetü'l-Hisâb*.³ Historians of science had studied these texts for their contents and place in the curriculum.⁴ Still, these works have to also be analyzed for their linguistic and stylistic aspects since the language is not only the medium of communication but also that of pedagogical transmission. Moreover, the content and style of the works written in Turkish are of importance for the analysis of the development of Turkish as a scientific language. In this study, our focus will be on one of the earliest examples of the genre, *Cāmī'ü'l-Hisâb*. Its author and contents will be briefly introduced and then, its linguistic and pedagogic aspects will be examined.⁵

1. On *Cāmī'ü'l-Hisâb*

We have very few information about the author of the *Cāmī'ü'l-Hisâb*. It was prepared by Yūsuf bin Kemāl el-Bursevî and completed in Muharrem 934 AH (October/November 1527). The author attests in the preface that “from some time, this humble servant Yūsuf bin Kemāl Bursevî kisses the exalted dust of his Excellency İskender Çelebi Efendi's –that his fortune continue!– feet (*hazret-i İskender Çelebi dāmet me'ālihu hazretlerinin hāk-i pāy-i şeriflerine bu bende-i naḥḥ Yūsuf bin Kemāl Bursevî bir niçe müddet yüz sürüyü*)”.⁶ One may deduce from this assertion that author was a clerk working under İskender Çelebi, a renowned *defterdar* (comptroller) during the reign of Suleiman the Magnificent.⁷ Famous Ottoman bibliographer Katip Çelebi confirms this information and adds that *Cevāmī'ül-Hisâb* (sic!) was written in Turkish.⁸ Most probably, the author was from the city of Bursa, former capital of the Ottoman State, famous for its educational institutions. Kepeci's *Register of Bursa* concisely informs that the author “was the son of a certain Kemal. He was a clerk during the reign of Suleiman the Magnificent and had a work known as *Cevami'ül-Hisab* (sic!).”⁹

OMLT enumerates seven copies of the treatise. During the research, two other copies were found. Here is its list in chronological order:

1- Süleymaniye: Lala İsmail, 288. Autograph. 120 folios, twenty-one lines each, written in *nesih* calligraphic style with diacritical marks (*harekeli*). Completed in Muharrem 934 AH (October/November 1527) by Yūsuf bin Kemāl Bursevî.

2- National Library: A.541. Autograph. 175 folios, thirteen lines each, *nesih* with diacritical marks. Completed in Safer 934 AH (November /December 1527). However, first eight folios are missing.

3- Fatih Millet Library: nr. 40. 113 folios written in *nesih* calligraphic style. Copied probably in the 11th century AH (1592-1688). The work is entitled as *Terceme-i Cāmī'ü'l-Hisâb*. Copyist's name is not mentioned.

4- Zeytinoğlu Provincial Public Library: nr. 1709/2 (303/2). Part of a collection of works bound in one volume, folios 60v-156r. Each page has twenty-one lines, with a plain *nesih*. Copied probably in the 11th century AH (1592-1688) by an unknown copyist.

5- Gazi Hüsrev Bey Library, R6930. 68 folios, twenty-six lines with *nesih*. Copied by Ahmed Nakkāş in 1115 AH (1703-1704).

³ Cf. Şermin Kalafat, “Anadolu (Osmanlı) Sahasında Yazılmış En Eski Tarihli Türkçe Matematik Risalesi: Mahmūd bin Kādī-i Manyās'ın A'cebü'l-Üccâb'ı-Hesap Bölümü, *Turkish Studies, International Periodical For the Languages, Literature and History of Turkish or Turkic*, Volume, 12/30, p. 243-298.

⁴ C. İzgi, *op. cit.*

⁵ This article is extracted from my PhD thesis submitted and defended in 2015.

⁶ Lala İsmail, 934: 2a/1-3.

⁷ Ekmeleddin İhsanoğlu -Ramazan Şeşen ve Cevat İzgi, *Osmanlı Matematik Literatürü Tarihi [OMLT]*, IRCICA Yay., II vols. İstanbul, 1999, vol. 2. p. 100; Bursalı Mehmet Tahir, *Osmanlı Müellifleri*, Matbaa-i Âmire, 3 vols., İstanbul, 1342, p. 309.

⁸ Kâtip Çelebi, *Keşfü'z-Zunûn*, Rüştü Balcı (trad.), Tarih Vakfı Yurt Yay., 2 vols., İstanbul, 2013, p. 513.

⁹ Kâmil Kepecioğlu, *Bursa Kütüğü*, Hüseyin Algül, Osman Çetin, Mefâil Hızlı, Mustafa Kara (ed.), 4 vols., Bursa, 2008, p. 264.

6- Museum of Topkapı Palace Library: H1991. 91 folios twenty-one lines with *talik*. Copied by a certain Mustafa in 1157 AH (1744-1745).

7- Cerrah Paşa Medical History Museum Library: 307: 68 folios, 25 lines with *nesih*. The date of the copy and the copyist's name are unknown.

8- Kastamonu Provincial Public Library: 37Hk3816/5, 112r-215v (104) folios, 15 lines with *nesih*. Copyist's identity is unknown.

9- Trinity College (Cambridge): Trinity R.13.11: 170 folios, 12 lines, *nesih* with diacritical signs. The date of the copy and the copyist's name are unknown.

In this article, the first two autograph copies were used. However, since the second (Milli Library A.541) manuscript lacks the eight initial folios, the first, which is Süleymaniye Lala İsmail 288, was taken as the main source and quotations are done from there.

2. Contents of the Treatise

1) First Chapter: On the forms and rules of multiplication, in five sections: The first section is on the multiplication of integral numbers. The second is about the multiplication of integral numbers with the decimals. The third is about the multiplication of decimal numbers with decimals. The fourth one is about the multiplication of integral numbers with the decimals and then the multiplication of the sum with an integral number. The last one is again a composite operation involving the multiplication of integral numbers with the decimals and then the multiplication of the sum again with another sum of a multiplication of integral numbers with the decimals.

2) The second chapter is on the decimals and their uses and comprehends calculations of metrological values such as *mūt*, *zirā*, *miskāl* and *kaṭṭār*.

3) The third chapter is about division and the operations related with it. It comprises six sections. The first section is on the division of integral numbers. The second is on the division of decimals. The third is about the division of decimal numbers by integral numbers. The fourth is about the division of decimal numbers by integral numbers by another decimal. The fifth is on the division of decimal numbers by integral numbers and then the division of the result again by the result of a division of a decimal number by an integral one. The last is about the divisions whose results are not an integral number but a fractional one.

4) The fourth chapter is on the common denominators and the relevant problems based on the multiplication and division of the fractional numbers, i.e., the proper fractions, compound fractions as well as the subtypes of the latter (discrete and repeating decimals).

5) The fifth chapter is on the division of inheritance. The author tells that he is particularly expert on this (35a/5-8) and explains the particularities and vicissitudes of the operation from his own experience.

6) The sixth chapter is about the rational numbers and comprehends calculations as well as problems regarding to them.

7) The seventh chapter is on the calculations of single and double derivatives and the relevant cases.

8) The eighth chapter is on square roots and their extraction.

9) The ninth chapter is about applied geometry and the relevant problems based on that matter. The angles, polygonal forms (triangle, rectangle, square, pentagon etc.) as well as the circle and the circular forms (semicircles, hemi circles, diameters, diametric ranges, arcs, crescent etc.) are treated here. Moreover, topics such as volume, height, distance as well as applied problems are discussed.

10) The tenth chapter is about equations. In other words, it is the equivalent of algebra and *mukābele* sections of the Ottoman mathematical texts and covers the so-called *meşā'īl-i sitte* (the six problems). The first three are the plain ones and the latter three are the compound ones.

3. The Author and the Interlocutor: Identity, Educational Level of the Student and His Relationship with the Teacher

As is the case in all levels of communication, in pedagogic communication, the subject matter, the power relationships as well as the cultural a/symmetries matter. In other words, the speaker's style depends on his/her interlocutor. Therefore, it is of importance to determine the identity and other basic proprieties of the interlocutor. It is important to identify to whom Yūsuf bin Kemāl is addressing to. The main interlocutors are seemingly Turkish speaking students.¹⁰ As the author specifies in the preface of his work, the target is mostly the beginners and not those who already are experts. (...*mübtedīler için beyān olunur yoksa ehl-i fażıl yanında ne miqdārı ola*).¹¹

Another constituent of the style is the educational level of the interlocutor. Implications, allusions and instructions depend on this decisive factor. Although it is not specified clearly, the contents of the work give some clues. *Cāmī'ü'l- Hisāb* starts directly with the operation of multiplication. In other words, elementary operations such as addition and subtraction are not treated. Then comes up integral numbers, decimals and goes with more complex problems which make us deduce that the interlocutor is not an absolute beginner.

3.1. How to Address the Interlocutor? A Stylistic Matter

In this kind of works, the explanations are mostly in passive voice. However, Bursevî uses more often than not the active and the lessons, examples as well as the problems are given both in first person plural and second person singular active. His style can be summarized as first the precision of the rule, then the question and at last the answer. When the question is formulated in second person singular, the answer is formulated in second person singular. (6b/3-5: *cümlesinüñ kıymeti ne miqdār olur darb-ı kaḫesiyle istihrāc etmek dilesen/ ... cümle idüp cevāb viresin* 7r/3 [what will be the sum if thou are to extact... Answer to this summarily]). When the question is formulated in first person plural, then the answer is in first person plural. In some cases, there are also some shifts: *eyle olsa beş dirhemi iki şey üzerine ziyāde eyledüñ yedi şey'i kāmil olur. Beş dirhem daḫı ma'ādil üzerine... ziyāde iderüz kim yigirmi üç dirhemdür yigirmi sekiz olur* [Thence, when you add two dirhams to five dirhams, that will amount to seven. When we add another five dirhams to the sum of twenty three, that will make twenty eight] (84 v15-85r /8). One may interpret these shifts from the second person singular to first person plural as an attempt to isolate the student in order to facilitate his concentration on the problem. And when the author passes to the plural voice, it is to assure his addressee, especially when the latter has to make some conclusions. Quite often, second person singular is used in easier cases and the plural in complex ones by the author.

4. Pedagogical Style of the Author

Today, mathematics is taught by several techniques such as plain demonstration, terminological, inventive, analytical, application ones etc. In the text, Yūsuf bin Kemāl uses some of them.

¹⁰ also see, İhsan Fazlıoğlu, "Osmanlı Döneminde "Bilim" Alanındaki Türkçe Telif ve Tercüme Eserlerin Türkçe Oluş Nedenleri ve Bu Eserlerin Dil Bilincinin Oluşmasındaki Yeri ve Önemi", *Kutadgubilig Felsefe-Bilim Araştırmaları*, 2003, p. 151-184.

¹¹ 2v/20-21.

4.1. Scenario Based Teaching

Scenario based teaching intends to inculcate the logic of the matter to be taught through putting the addressee in concrete contexts.¹² Yūsuf bin Kemāl makes abundant use of this technique as cited below:

“mes’ele-i āḥar iki ḥurmā ağacı olsa bir āb kenārında birinüñ ṭūlī yigirmi beş zirā’ olsa ve birinüñ ṭūlī yigirmi zirā’ olsa, birbirine ıraklığı altmış zirā’ olsa, bu ḥurmā ağacının depesinde 80b/16-19. birer kuş olsa, bu şuda balık gözedür olsalar, ikisi bile bir balık görüp berāber uçsalar bir ḥaṭṭ-ı müstaḥḥim üzere mü’eyyed ile bu balığa yitişseler, bu balık da şu içinde bir ağacdan bir ağaca bir ḥaṭṭ-ı müstaḥḥim farz eylesen, anuñ üzerinde olsa bilmek istesen kim ol kuşlar her birisi ne miḳdār yir uçarlar ola, yā balık olduğı yirden ağac dibine dek ne miḳdār yirdür.” [Another riddle. Let’s assume that there are two palm trees – one being twenty five ziras of height and the other twenty– alongside a pond whose distance to one another is sixty ziras. Above each of it there is a bird who watches out a fish. Both see it and prey on it simultaneously following a direct line. At that moment the fish makes the distance of two trees. Now find out the distances that both of the birds have made and the distances from the fish’s initial position to the roots of the threes].¹³ The question is given with lavish detail. There are many narrative elements which are not of mathematical use *per se* as the details about the tree. Moreover, the initial setting enables the author to ask two separate questions. The narrative sequencing allows the author to develop these questions further without vexing the students.

Here is another example with an engaging narration. *“eger ‘Amurun iki biñ üç yüz akça mālī olsa, bir metā’a virse; ba’dehū otuz gün geçdükden sonra Zeyd gelse iki biñ altı yüz akça ol daḥı getirse, birbiriyle şerīk olsalar; ba’dehū on günden sonra Bekir daḥı gelüp üç biñ akça sermāye ol daḥı getirse şerīk olsalar; ba’dehū diger Bekir daḥı yigirmi günden sonra gelüp iki biñ sekiz yüz akça ol daḥı getirse, cümle bu dört nefer kimesne şirket ṭarīḳiyle bir yıl ticāret idüp cümle biñ altı yüz akça fāyide hāşıl itseler; ammā ticāretleri şol şart üzere olsa ki fāyidelerin her birinüñ gününe 36b/5-16 ve mālına göre kısmet itmek dileseler; bu taḳdīrce her birinüñ ḥişşesi ne miḳdār vāḳı’ olur bilmek dilesen...”* [Let’s suppose that ‘Amr has some goods whose worth is two thousand and three hundred aspers. Then he sells some of it. A month later, Zayd comes up with a capital of two thousand and six hundred aspers to form a partnership. And then Bekr arrives twenty days afterwards with a capital of three thousand aspers. Following this, another Bekr comes up with two thousand and eight hundred aspers. Within this partnership, they make their trade during a year and make a profit of a thousand and six hundred aspers. The founding principle of the enterprise was the distribution of the profit according to the sum and daily involvement of each of the capital supplier. Let’s now find the profit of each trader].¹⁴ The question is laid out in complex manner. A detailed narration enables an easier focus on the problem.

4.2. Teaching by definition

In teaching by giving definitions first, the teacher explains every term in advance. By so doing, the student learns their use within the proper configuration. In this technique, both the terms that will be of use and those which will not be, given together.¹⁵ In the sections where the

¹² see, Mehmet Mustafa Toker, “Aktif Öğrenme”, *Üniversite ve Toplum Dergisi*, III/1, 2003, p. 7-19; Banu Yaman, “Senaryo Tabanlı Öğrenme Yaklaşımına (Stöy) Dayalı Eğitimde Drama Yönteminin İlköğretim Beşinci Sınıf Öğrencilerinin Okuduğunu Anlama Başarılarına Etkisi”, *Ç.Ü. Sosyal Bilimler Enstitüsü Dergisi*, XIV/ 2, 2005, p. 485-492; Esen Ersoy and Neşe Başer “İstatistik ve Olasılık dersinin Senaryo İle Öğretim Süreci Sonunda Öğrencilerin Eleştirel Düşünme Eğilimlerindeki Değişim”, *Ondokuz Mayıs Üniversitesi, Eğitim Fakültesi Dergisi*, 33, 2014, p.1-15.

¹³ 81a/1-6

¹⁴ 37a/1-3.

¹⁵ M. Toker, *op. cit.*, s. 12.

author informs about the science of *misaha* (measurement of distance and space), definitions of different terms are given. Often, the author gives detailed explanations without initially giving examples and making allusions. Here is an example: “[c]ezir aña dirler ki bir ‘adedi gendü nefesine urasañ ol ‘adede cezir dirler ve darbdan hâşıl olan ‘adede meczûr dirler ve murabba‘ dañı dirler ve māl dañı dirler.” [jazir is the multiplication of a number with itself. The result is called the *majur*. It is as well called as *murabba* and even *māl*].¹⁶

4.3. Teaching by rule

Teaching by giving the rules first implies the learning by heart of some of the operations. Even if nowadays this technique is less used, still some operations are still inculcated by this manner such as the extraction of square root as well as some complex operations in algebra.¹⁷ There are many instances in the text where this method is applied. Here, we will give the example of the extraction of the square root: “faşl-ı sâmin muḍalli‘ât beyânındadır bir niçe kısım üzerine beyân olunur. Kısım-ı evvel cezr-i mantık beyânındadır. Cezir aña dirler ki bir ‘adedi gendü nefesine urasañ ol ‘adede cezir dirler.” [Chapter eight: This is about how to extract square roots. It will be explained according to the cases. The first is about the extraction of root of rational numbers. A square root operation involves the multiplication of the same numbers. The result is a perfect square.]¹⁸

5. Use of Synonyms and Deductive Forms

5.1. Teaching the Terms with Their Synonyms

One of the difficulties that the author is affronted with is the fact that his students are monolinguals. The latter probably know only Turkish while the terms are in Arabic. Often the author explains the term starting with an *i.e.* (ya‘nî) or eventually, with a synonym in Turkish.

Here is an example: “mālî kâmil kıılıruz; ya‘nî şî ḥums mālî dü ḥumsa ilḥāk iderüz tã kim bir māl olur ve mālî ziyâde eyledüğümüz miḳdârı muḳârinine dañı ziyâde iderüz; ya‘nî on bir cezri on altıya **kataturuz** tã kim...” [We complete the square in other words we add up the three fifth to two fifth so as to it make a square. And then, we increase *i.e.* that is to add up the square with another one].¹⁹ Here, the Turkish verb “*katmak*” designs both to make perfect (*kâmil kıılmaḳ*) and add up (*ilḥāk itmek* as well as *ziyâde itmek*).

5.2. Explaining the rule by making use of the “lâzım” and “gerek” formules

The words “lâzım” and “gerek” in Turkish are synonyms and both design a necessity. We see in the text their use in many instances generally for two different situations:

“Aşıl mālî sekiz farz eyledük; pes sekiz dañı ilḥāk olunduḳda on altı oldu içinden üç dirhemin aldūḳ, on üç bâḳî ḳaldı; ba‘dehū ol miḳdār dañı ilḥāk eyledük, yigirmi altı olur; içinden on dirhemin aldūḳ, bâḳî on altı dirhem ḳaldı; ba‘dehū bir ol miḳdār dañı ilḥāk eyledük, otuz iki oldu; pes yedi ‘adedin giderdük, bâḳî yigirmi beş ḳalur; bize ḥōd toḳuz ḳalmaḳ **gerekidi** kim şaḥîḥ olaydı; pes ḥaṭâ on altı vâḳî ‘oldı zâyiddür.” [We had assumed that the capital is formed with eight dirhams. When we had added another eight dirhams the sum amounted to sixteen. When we had subtracted three dirhams, we had thirteen in total. After adding to this another thirteen units, we had twenty six dirhams. After subtracting ten dirhams, our capital shrank to sixteen. When we added as much as this, we had thirty two dirhams. After deducting seven, we arrived at the sum of twenty five. But normally we **had to** have nine in order to have a valid operation. We have now a superfluous sixteen. 48a/6-14]. In this example, the author

¹⁶ 53a/3- 6.

¹⁷ M. Toker, *op. cit.*

¹⁸ 53a/7-11.

¹⁹ 89b/2-5.

makes use of the “*gerek*” so as to accentuate the necessary condition and as such, accentuates the absoluteness.

There is another use of this formula: “*Pes cevāb virürüz ki ikinci yāķūtun bahāsı ki bir şey tutmuşıduñ yedi yüz yigirmi dinār olur; lāzım geldi ki bahā-yı yāķūt-ı evvel altı yüz kırk dinār ola ve üçüncü yāķūtun bahāsı sekiz yüz kırk dinār olur; çün bu üç kıymeti cem eyledüñ iki biñ yüz dinār olur ve her mes’ele ki bu cins vāķı’ ola bu kıyās üzerine iħrāc olına.*” [We can respond in this manner: You had taken the value of second ruby as x. That makes seven hundred and twenty dinars. The first **must have been** worthy of six hundred and forty dinars and the third, eight hundred and forty. Their sum makes two thousand and a hundred dinars. This sort of problems may be thus resolved in comparison to this particular one. 116b/3-9.] Here the use of ***lāzım geldi ki*** implies the result which is required. In other words, he makes a deductive implication.

6. Coherence and Cohesion

One of the basic pedagogic rules is the clarity in the exposition. Another one is the exemplification by asking the similar questions in multifarious manners. The author is reminiscent of these rules and applies them regularly.

6.1. Use of Conjunction Connectives in the Explanation

Often the author makes use of explanatory formulas. Here are some cases:

“*Pes iki yüz elli zāyid olur haṭā vāķı’ oldı ve s̱ānīye semen-i elmās sekiz yüz elli dirhem farz eyledüñ ol taķdīrce semen-i yāķūt yedi yüz on altı dirhem ve s̱ülüsān dirhem olur ve semen-i la’l altı yüz kırk bir dirhem olur ve s̱ülüsān dirhem olur anuñıçün kim semen-i yāķūtun nısfı üç yüz elli sekiz dirhem ve s̱ülüs dirhem olur biñden tarh idecek altı yüz kırk bir dirhem ve s̱ülüsān dirhem olur çünkim semen-i la’l bu miķdār farz olundu.*” [As such, two hundred and fifty exceeds. But this is a false conclusion since you had assumed the price of the second diamond as eight hundred and fifty dirhams. This makes the value of the ruby 716+1/6 dirhams and that of the other gem, 641+1/6 dirhams. *Because of this*, half of the price of the ruby is 358 +1/3 dirhams. If you are to subtract it from thousand you will have 641+1/6 dirhams *because* the other gem’s price was attributed to this value 52a/1-9.] Here we see that the author uses many explanatory connectives such as *anuñıçün kim*, *çünkim*.

6.2. The use of opposing conjunction in order to give possibilities and facilitate the answer

The author uses opposing connectives in order to give possibilities and facilitate the answer. As such, he shows the students the means through which they can solve problems similar in nature. Here is an example: “*eger iki haṭṭı birbirine ulaşacak olursa ammā istiķāmet üzerine degül ol vaķt ol şaṭḥ ki iki haṭṭı ortasında vāķı’ olur.*” [When two lines coincide but not in the same plain thence their axe is in their middle.” 72a/13-15]. The author uses here the connective “but” in order to accentuate the point made.

6.3. Use of Conditionals

While teaching, Bursevî uses abundantly the conditionals. The conditional suffix “-sa” is widely utilized with if or when clause. In some other cases, he makes use of “*gerekse...gerek*” formula. By doing so, he enables the interlocutors to seize the means through which they can solve problems similar in nature: “*eger bir şekil olsa dıl’luları müsāvī ammā bilmesey ki ḵāyimü’z-zāviye midür yā murabba’ midür yoksa mu’ayyen midür her iki kuṭrını ölçesin eger mütesāvī olacak olursa murabba’ olur eger mütesāvī olmazsa mu’ayyen olur ḵā’idedür.*” [When there is a polygonal form and you do not know if it is a square or a rectangle. You have to

measure the longitudes. *If* they are the same, it is a square and *if not* it is a rectangle. That is a rule 75b/15-19].

7. Order and sequence

One of the crucial points while teaching mathematical operations is the order and sequence of operations. This is often made with the proper usage of sequence connectives which directs the ordering (priority, generality etc). In this, the proper ordering of the question is equally important because this hints as well to the order of the operations to be done. First part of the question refers to the first operation, the second part to the second operation and so forth.

Here is an example: “*eger bir top olsa gümüşden, vezni üç yüz seksen beş dirhem olsa; bir top dağı olsa altın ile gümüşden; büyüklükde evvelki topun aynı olsa, vezni dört yüz yetmiş beş dirhem olsa ol taḳdīrce sonrağı topun altını ne miḳdār ve gümüşü ne miḳdār olur.*” [Let’s assume that there is a silver ball which weighs three hundred eighty five dirhams. And if there another ball made with golden and silver of the same size which weighs four hundred and seventy five dirhams. What is the latter’s gold and silver ratio? 111b/9-13.] The author first gives the value of the first ball and then, the second. The operation will be made by making an equation.

Here, the author reminds first the rule to be kept in mind: “...*ba’dehū sānīye’l- ‘adedde cümle eyledüğün a’ḳādı gine mezbūr altı ‘adede ḳarb idesin; ne miḳdār ‘aded ḥāşıl olursa aşıl mālдан ol miḳdār a’ḳād tarḥ idesin; eger bāḳī ‘aded ḳalmayup tamām vefā iderse ol vaktin māl-ı ka’b-ı munṭaḳ diyessin ve eger bāḳī ‘aded ḳalursa māl-ı ka’b-ı āşamm diyessin.*” [Afterwards, you have to multiply the numbers in the second set with the aforementioned six numbers. And then you subtract the result from the initial total sum. If there remains no resting number, then you will have an equivalent of fifth degree of rational numbers; if not, than that of a fifth of irrational numbers 70b/2-9]. The author provides and teaches a rule through solving a problem. While passing to a new operation, he uses a sequencing connective (*ba’dehū*). While designing another operation, he uses another sequencing connective (*gene* [again]) to denote this one. At the end of the case, to point at two probable outcomes, he uses the *eger* and *eger* (if...if not) formula.

Here is another example in which sequencing is carefully made: “[*Meşelā iki māl iki mālā ḳarbdan dört mālul-māl ve illā iki cezir illā iki cezir dört māl illā iki cezir illā iki māl illā dört ka’b ḥāşıl olur bu mes’ele de gereḳdür ki evvel bir def’a 84a/19-21 zāyidi zāyide ḳarb ide ikinci bir kerre nāḳış nāḳışa ḳarb ide üçüncü bir kerre nāḳışa zāyidi ḳarb ide dördüncü bir kerre nāḳış zāyide ḳarb ideler tā kim bu cümleñ murabba’ı dört māl’ul-māl ve dört māl illā sekiz ka’b olur.*” [For instance, the multiplication of two x^2 with another two x^2 , yields to four x^4 minus two x (as a root) minus two x and four x^2 minus two x minus two x^2 minus four x^3 . So in this sort of operation, it is required first to multiply the positive values, secondly the negative ones, thirdly the positive with the negative one and finally, the negative with the positive one. As such, the square of the sum makes four x^4 minus four x^2 minus eight x^3 84b/1-4.] Here each operation is denoted with the proper sequence as first, secondly etc.

8. Use of Compound Verbal Forms

The author does not only notify the probabilities and sequencing with the connectives but also by the use of periphrastic forms.

8.1. Periphrastic Forms

a) {-miş} + ol-: Zeynep Korkmaz defines this compound verbal structure as the precision of a previous act.²⁰ In our text, when this structure is deployed, it denotes both the problem and

²⁰ Zeynep Korkmaz, *Türkiye Türkçesi Grameri-Şekil Bilgisi*, TDK Yayınları, Ankara 2003, p. 151.

the result. In a detailed problem about three couriers who set to their goals in differing moments and who advance in different pace, the result is explained by using this structure: “*pes cevāb viresinkim peyk-i şānī peyk-i evvele biñ dört yüz toköz kām ve bir subu‘ kāmda yetişmiş olur; bu peyk-i şānīnūñ kāmıdır bu taqdīrce lāzım olur ki peyk-i evvel iki biñ altı yüz kırk iki kām ve bir subu‘ kām gitmiş olur; her gāh ki bu aşıl mes‘ele vākı‘ ola bu kıyās üzere istihrāc idesin.*” [Answer to this in this manner: The second courier attains the first in 1409 +1/7 time unit. This is the second courier’s pacing time. You can deduce the first courier’s pace according to this which is 2642 +1/7. Each time when you will have this kind of problem, solve it thus by comparing to this pattern. 105b/1-7]. Here with the use of this compound structure the event is generalized in the aorist form. Rather than sequencing, it denotes a certainty.

b) {-(U)r} + ol-: According to Korkmaz, this compound structure denotes a “habitude”.²¹ In our case, it denotes in a similar way, regularity:

“*mes‘ele-i āḥar iki ḥurmā ağacı olsa bir āb kenārında birinūñ ṭūlī yigirmi beş zirā‘ olsa ve birinūñ ṭūlī yigirmi zirā‘ olsa birbirine ıraklığı altmış zirā‘ olsa, bu ḥurmā ağacınūñ devesinde 80b/16-19. birer kuş olsa, bu şuda balık gözedür olsalar ikisi bile bir balık görüp berāber uçsalar bir ḥaṭṭ-ı müstaḳīm üzere mü‘eyyed ile bu balığa yitişseler, bu balıkda şu içinde bir ağacdan bir ağaca bir ḥaṭṭ-ı müstaḳīm farz eylesen, anuñ üzerinde olsa bilmek istesen kim ol kuşlar her birisi ne miḳdār yir uçarlar ola yā balık olduğı yirden ağac dibine dek ne miḳdār yirdür.*” 81a/1-6. (For the translation of this quotation, See 4.1.a.). In this case, the author implies with the use of this compound structure that the birds in question are of piscivorous nature and that they do this watch as a habitude. Here we can even say that the author informs the students about daily zoological facts.

8.2. Some compound verbal forms with { -maḳ } + bul- and gör-

The author sometimes makes use of compound verbal forms with “bul- and gör- verbal forms in order to hint at necessary operations or conclusions. Here is an example: “... *muwāfaḳat talep eyledūñ nişf olmaḳ bulundı/göründi.*” [You have wanted coherence, thence the half]. Here is a concrete case: “*Pes kırk ikiyle sekizūñ maḥreci şümündür muwāfaḳat taleb eyledūñ nişf olmaḳ bulundı.*” [The coefficient of forty two and eight are one eightieth. You have wanted coherence, thence the half. 32a/2-4]. In the following case (32a/7-8), the author uses *olmaḳ göründi* form. Each of these cases designates a rule.

8.3. Compound verbal structures with to be form

a) Noun +ol- + {-mıṣ}+ ol- : When compared to a similar but a more simple verbal structure with {-mıṣ ol- }, this refers to an operation previously accomplished. Here is an example: “[O]l taqdīrce kısıp-ı evvel yigirmi dirhem ve rub‘ dirhem olur ki şeyi farz olınmış ola.” [According to this, first part is constituted by twenty dirhams and a quart assuming that x is predefined in advance. 94b/1-2.]. In other words, by resorting to this Formula, the author implies that the value of x is already assumed or known by heart.

b) Noun + ol- + {-AcAḳ}+ ol- : According to Korkmaz, the compound verbal structures with {-AcAḳ ol-} designate an intention.²² Here {- AcAḳ} with an “ol-” in the aorist with a desiderative, makes the intention to evolve to a firm intention, thence to a condition. Here is an example: “[E]ger ikisi bile zāyid yā nāḳıṣ olacaḳ olursa ziyāde olandan eḳallın tarḥ idüp bāḳī ḳalan ‘adedi yazup pes bāḳī-yi evvel diyesin.” [If both of them happens to be positive or negative, you are subtract the minor from the major and write the result as the first one. 47a/11-13]. Here is another example: “[E]ger āḥir ḥānenūñ üzerinde sıfır vākı‘ olacaḳ olursa üzerine bir ‘aded bulup yazup aḡa göre kısmet idesin” [If in the last digit, there is a zero, you have to

²¹ Korkmaz, opt. cit., p. 803.

²² Korkmaz, opt. cit., p. 806.

add up a rational number and divide according to that. 53a/17-18]. As it is evident from these, the rules are often précised by using this grammatical structure.

9. Use of the Poetical Style and Patterns

In the Ottoman Mathematical Works, and more generally in the madrasa curriculum, use of poetry and poetic patterns are common as a mnemotechnic method. These poems are often in Persian and Arabic. In *Cāmī'ü'l- Hisāb* there are three poems in Persian but there is a Turkish one as well:

(1) *ne buyururlar işbu mes'elede kâşif-i müşkilât-ı ehl-i kalem* [See what they offer in this matter, men of the pen the revealers of the difficulties]

(2) *aldı bir hoca üç 'aded gevher* [The Hodja took three gems]

(2a) *kıymeti iki biñ iki yüz dirhem* [whose value in total is two thousand and two hundred dirhams]

(3) *gel kerem kıl aya daķık-i fikir* [Give us a help oh! the intelllligent man]

(3a) *her birinün bahāsın eyle raķam* [Write down the value of each of them]

(4) *didi ol hōca-ı rumūz-gūşā* [Said the Hodja the solver of mysteries]

(4a) *bu su'ālün cevābın mübhem* [The answer to this is a bit confuse]

(5) *evvelün kıymetine s̄ān̄mün* [If to the value of the first]

(5a) *nışf-ı kıymeti eger olursa zamm* [Is added the half of the second]

(6) *biñ olur l̄k̄ī s̄ān̄ kıymetine* [The sum is one thousand]

(6a) *s̄ülüs s̄ālīs eger olursa münzamm* [And when one eighteenth is added to the value of the second]

(7) *hem-çü-nān girü biñ olur ammā* [The sum is one thousand yet gain but]

(7a) *s̄ālīs rub ' evvelki olsa münzamm* [And when one twelfth is added up to the first]

(8) *eger olursa biñ olur b̄ī-şekk* [The sum is one thousand yet]

(8a) *bunı fīkr gūde böyle dut muħkem* [Know it thus firmly]

(9) *her ki māhirdür bu fenn içrekim* [The one who is an expert in this field]

(9a) *müşkilün ĥall ider kıomaz mübhem.* [Solves every problem and leaves nothing vague] 51b/1-9a.

In these verses, the author narrates a puzzle where there is a problem and a way to resolve it. In the last three verses, “Know it thus firmly: The one who is an expert in this field Solves every problem and leaves nothing vague”, exhorts the student to be versed in subtle problems by mastering the technics of the science of Mathematics.

Conclusion

After this scrutiny, we may infer that the author of this mathematical text's style depends mostly upon his primary objective which is to give someone the knowledge of basic rules of mathematics and to train and instruct his students in that subject. However, his text is not dry and boring, at all. It is accurate and concise in order to inculcate the rules and to make figure out the problems. The author, Bursevî, uses mainly three pedagogic techniques: teaching with definition, rule and scenery. He even renders the problems in versified, poetical format in order not only to entertain but also to facilitate the memorisation. Both in explanation and exemplification, he uses basic grammatical and stylistic patterns. He also diversifies the examples as well as cases. In other words, he redefines in accordance with the context. He enriches his explanations with pedagogic plots, descriptions and allusions.

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