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Gyroless Nanosatellite Attitude Estimation in Loss of Sun Sensor Measurements

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Abstract: The paper introduces gyro-free attitude estimation methods for sun and eclipse phases. First of all, traditional and nontraditional attitude estimation approaches are investigated relying on sensor measurements from magnetometer, and sun sensor. For the nontraditional approach, vectors are placed in Wahba's problem and the loss function is minimized by Singular Value Decomposition (SVD) method. The coarse attitude information and their variances from SVD are used in the extended Kalman filter (EKF) as linear measurements in order to estimate the satellite's attitude and angular rate. With using the same filtering parameters and initial conditions in traditional EKF, a comparison is made for the sun phase of the orbit. However, for the eclipse period (in loss of Sun sensor measurements), two additional methods are introduced as the prediction method, and traditional EKF based on only magnetometer measurements. An accuracy performance analysis is performed, for different phases, a switching algorithm is used for better attitude and rate estimation.

Keywords: nanosatellite, eclipse, gyro-free, attitude estimation.

1. INTRODUCTION

Several methods of attitude estimation algorithms for attitude control system have been developed for small satellites with precision pointing requirements of the mission. Small satellite missions are under budget restriction; therefore, require reliable and inexpensive attitude determination hardware such as coarse sun sensors and magnetometers.

Kalman type filters are commonly used in order to estimate the satellite's attitude and angular rates (Morelande & Garcia-Fernandez, 2013; Psiaki et al., 1990; Sekhavat et al., 2007). The conventional approach to estimate the rotational motion parameters is to use extended Kalman filter (EKF) based on the nonlinear models of satellite motion and measurements, with Gaussian noises (DaForno et al., 2004).

Using at least two vector observations, attitude angles can be determined using deterministic single-frame methods such as q, QUEST, Singular Value Decomposition (SVD) (Cilden Guler et al., 2017; Markley & Mortari, 2000). The vector observations from these methods can be directly used in the Kalman filters with their covariance and they are treated as linear measurements in the filter. This way, the filter gains an adaptive structure because of having variational covariance in time depending on the measurements (Hajiyev et al., 2016; Hajiyev & Bahar, 2003; Markley & Crassidis, 2014; Vinther et al., 2011). The preprocessing in single-frame methods involves the minimization of Wahba's loss function (Wahba, 1965). In (Cilden-Guler et al., 2017), a comparison of minimization methods concludes that the singular value decomposition (SVD) and q methods are the most robust for single-frame attitude estimation methods. The methods based on linear and nonlinear measurements can be called nontraditional and traditional methods respectively. A review on the methods for gyroless attitude determination system of a small satellite is performed in (Hajiyev & Cilden-Guler, 2017).

In (Cordova-Alarcon et al., 2015), a method is proposed based on Gauss-Newton and EKF using sun and magnetic vector observations in order to provide three axis attitude estimation in both sun and eclipse periods. The authors concluded that the obtained results show a good performance of the attitude propagation process even in the eclipse phase. However, even the authors use a nontraditional structure for the EKF, the covariance of the measurements are taken as constant. In (Hajiyev et al., 2017), prediction method is suggested for the eclipse period besides a nontraditional method in adaptive scheme for the sun-phase but it is concluded that the accuracy of the results from the algorithm is not sufficient if the eclipse period is long-term.

In this study, several attitude estimation methods are considered for nanosatellite's rotational motion. In the first stage, in the sun-phase, two filters based on EKF with traditional and nontraditional approaches are taken into account for a nanosatellite having magnetometer and sun sensor. For the second stage, due to the absence of the sun measurements in eclipse, two additional methods are proposed to maintain the attitude and rate estimation during this period. The paper structure is organized as follows. Section 2 is for a brief information on the mathematical models for the vector observations. Section 3 and 4 describes the EKF based on nonlinear and linear measurements respectively. Then, the prediction method for satellite's rotational motion is presented in Section V. The analysis and results are given in Section 6. As the final part of the paper, the conclusion remarks are given in Section 7.

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2. MATHEMATICAL MODELS FOR THE VECTOR OBSERVATIONS

Magnetometer and sun sensor measurements are used in this paper as the vector observations. For this purpose, mathematical models of the magnetic field and the sun direction vectors are introduced. For obtaining the magnetic field vector $\left(B_{IGRF}\right)$ in the orbital frame in nT, the most updated International Geomagnetic Reference Field (IGRF-12) model is used with the 3 position data and the time variable for every instant time. The IGRF used in the paper is 13th degree and updates its coefficients every 5 year (Thébault et al., 2015). To be consistent with the sun direction vector, magnetic field vector is normalized; and therefore, unitless

$$B_o = \frac{B_{IGRF}}{\left\| B_{IGRF} \right\|} \tag{1}$$

(1) shows the direction of magnetic field vector at an instant time; therefore, the values of each component are changing between ± 1. In order to simulate the three-axis magnetometer vector observations $\left(B_m\right)$ in body coordinates, magnetometer measurement noise $\left(v_B\right)$ is assumed to have a zero-mean Gaussian distribution and added on the transformed magnetic field model vector $\left(B_o\right)$ using the transformation matrix from orbit to body coordinates $\left(A\right)$ and described as,

$$B_m = AB_o + v_B \tag{2}$$

The unit sun direction vector $\left(S_{ECI}\right)$ can easily be obtained in Earth Centered Inertial (ECI) frame using only the Julian Day (Vallado, 2007). Then, it can be transformed into the orbital frame $\left(S_o\right)$ using the orbital parameters and can be denoted as S_o. Similar to the magnetic field observations, three-axis sun direction vector observations $\left(S_m\right)$ in body coordinates, sun sensor measurement noise $\left(v_S\right)$ is assumed to have a zero-mean Gaussian distribution and added on the transformed sun direction model vector $\left(S_o\right)$ using the transformation matrix from orbit to body coordinates $\left(A\right)$ and described as,

$$S_m = AS_o + v_S \tag{3}$$

3. NONLINEAR MEASUREMENTS BASED EKF

As it is discussed in the introduction part, EKF can be used in two different forms as linear and nonlinear measurement based. In this section, the traditional EKF, which is based on the nonlinear measurements, is introduced. The satellite's rotational motion about its mass center is formulated using the discrete-time nonlinear state space model

$$x\left(k+1\right) = f\left[x\left(k\right)\right] + w\left(k\right) \tag{4a}$$

$$z(k) = h\left[x(k)\right] + v(k) \tag{4b}$$

where $f\left[\cdot\right]$ and $h\left[\cdot\right]$ are the nonlinear dynamic and measurement functions respectively, $x(k)$ is the n dimensional state vector at time k, $w\left(k\right)$ is the zero-mean Gaussian noise with covariance $Q\left(k\right)$, $z(k)$ is the d dimensional measurement vector, $v\left(k\right)$ is the zero-mean Gaussian noise with covariance $R\left(k\right)$. It is assumed that both noise vectors $w\left(k\right)$ and $v\left(k\right)$ are linearly additive Gaussian, temporally uncorrelated with zero mean,

$$E\left[w(k)\right] = E\left[v(k)\right] = 0, \ \forall\, k \tag{5}$$

Filter algorithm based on the described system and measurements in (4) can be given. The approximations in the prediction and update stages of the filter can be found based on the EKF (Psiaki et al., 1990). The estimation value can be found as,

$$\begin{aligned} \hat{x}(k+1) &= \hat{x}(k+1/k) + K(k+1) \\ &\quad \times \left\{ z(k+1) - h\left[\hat{x}(k+1/k)\right] \right\} \end{aligned} \tag{6}$$

The extrapolation value from the dynamic function can be found as,

$$\hat{x}(k+1/k) = f\left[\hat{x}(k)\right] \tag{7}$$

Filter-gain of the EKF is,

$$\begin{aligned} K(k+1) &= P(k+1/k)H^T(k+1) \\ &\quad \times \left[H(k+1)P(k+1/k)H^T(k+1) + R(k) \right]^{-1} \end{aligned} \tag{8}$$

where $H(k+1) = \dfrac{\partial h[\hat{x}(k+1/k)]}{\partial \hat{x}(k+1/k)}$ is the partial derivatives of the measurement function with respect to the states.

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial f[\hat{x}(k)]}{\partial \hat{x}(k)} P(k/k) \times \frac{\partial f^T[\hat{x}(k)]}{\partial \hat{x}(k)} + Q(k) \tag{9}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = \left[I - K(k+1)H(k+1) \right] P(k+1/k) \tag{10}$$

The filter expressed by the formulas (7) - (10) is called the EKF based on traditional approach. Here, all the sensors are assumed to be calibrated; therefore, there is no bias on the measurements.

The state vector considered in this paper is,

$$x(k) = \left[\varphi(k) \quad \theta(k) \quad \psi(k) \quad \omega_x(k) \quad \omega_y(k) \quad \omega_z(k) \right]^T \tag{11}$$

where $\varphi(k)$, $\theta(k)$, $\psi(k)$ are the roll, pitch, yaw angles, and $\omega_x(k)$, $\omega_y(k)$, $\omega_z(k)$ are the x, y, z axes angular velocity components respectively. For the magnetometer and sun sensor based EKF, the nonlinear measurement vector $z(k)$ shown in (4b) is composed of magnetometer and sun sensor as,

$$z(k)=\begin{bmatrix}B_m & S_m\end{bmatrix}^T \quad (12)$$

For only the magnetometer based EKF, the nonlinear measurement vector $z(k)$ shown in (4b) is composed of only the magnetometer as,

$$z(k)=B_m \quad (13)$$

Related measurement matrix $H(k+1)$ and the measurement error covariance matrix $R(k)$ are formed depending on the specific application.

4. LINEAR MEASUREMENTS BASED SVD/EKF

Nontraditional approach uses a pre-processing step in order to obtain the linear measurements. In this paper, SVD as one of the single-frame methods is used because of its robustness (Cilden-Guler et al., 2017). Once, the SVD processed the measurements, and obtained the coarse attitude angle measurements and related covariance; EKF can use those variables which are linear attitude measurements $z(k)$ and their covariance $R(k)$. The filter is called SVD/EKF which is based on the linear measurements and it is able to estimate the state vector given in (11) as the attitude angles and the angular velocities.

SVD/EKF based on the linear measurements is applied for the same system given in (4a). The new linear (Euler angle) measurements from the SVD by processing the magnetometer and sun sensor measurements can be described as,

$$z(k)=\begin{bmatrix}\varphi(k)+v_\varphi(k)\\ \theta(k)+v_\theta(k)\\ \psi(k)+v_\psi(k)\end{bmatrix} \quad (14)$$

where $\varphi(k)$, $\theta(k)$, $\psi(k)$ are the attitude angles determined by SVD method, $v_{(\cdot)}(k)$ is the measurement noise of the attitude angles. The measurement error covariance matrix is calculated for every step in the SVD, using this variant $R(k)$ from SVD method makes the filter adaptive. Therefore, there is no need to define a constant matrix or design an adaptive structure for the measurements in SVD/EKF method.

As the measurements are linear, the measurement matrix will be a constant matrix defined as,

$$H=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix} \quad (15)$$

The equation of the estimation value defined in (6) can be replaced for the SVD/EKF as

$$\hat{x}(k+1)=\hat{x}(k+1/k)+K(k+1)\times\{z(k+1)-H\hat{x}(k+1/k)\} \quad (16)$$

The extrapolation value from the dynamic function can be obtained using (7). The filter gain of the SVD/EKF is,

$$K(k+1)=P(k+1/k)H^T\times\left[HP(k+1/k)H^T+R(k)\right]^{-1} \quad (17)$$

where $R(k)$ is the covariance matrix of the measurement noise, which has diagonal elements built of the variances of attitude angle measurement noises.

The covariance matrix of the extrapolation error can be found using (9). The covariance matrix of the filtering error is,

$$P(k+1/k+1)=\left[I-K(k+1)H\right]P(k+1/k) \quad (18)$$

The filter expressed by the above formulas is called the SVD/EKF based on nontraditional approach. Here, all the sensors are also assumed to be calibrated as it was for the traditional EKF method.

5. PREDICTION OF THE SATELLITE'S ROTATIONAL MOTION PARAMETERS

Besides the estimation techniques for the attitude angles and angular velocities of the nanosatellite, prediction method can be used especially in case of any malfunction or absence of the measurements. In this study, nanosatellite has sun and eclipse phases while orbiting around Earth. For these two phases, different methods are evaluated in the sense of their accuracy. Methods used for attitude and rate determination of the nanosatellite in which the phase is appropriate are discussed in the simulation results sections for each phase.

The prediction algorithm characterizing the satellite's rotational motion about its center of mass can be derived for nonlinear and non-stationary system as,

$$x(k+1)=f\left[x(k)\right] \quad (19)$$

where $x(k)=\begin{bmatrix}\varphi(k) & \theta(k) & \psi(k) & \omega_x(k) & \omega_y(k) & \omega_z(k)\end{bmatrix}^T$ is the state vector (11). Considering the parameter estimation error small, solution of the (19) can be expressed as

$$x(k)\approx\hat{x}(k)+\sum_{l=1}^{g}\frac{\partial f}{\partial\left(\Delta x_l(k)\right)}\Delta x_l(k) \quad (l=1,...,g) \quad (20)$$

where $\hat{x}(k)$ is the solution based on the parameter estimate values of $x(k-1)$ vector. $\Delta x_l(k)$ is the increment of the state parameters. In the prediction of the parameters of the rotational motion of the satellite, the changes in the external moments that affect the satellite should be taken into account as,

$$x^{pr}(k+1)=\hat{f}\left[x^{pr}(k),\, N^{pr}(k)\right] \quad (21)$$

Here, $x^{pr}(k)$ is the predicted values of the states, $N^{pr}(k)=\begin{bmatrix}N_x^{pr}(k) & N_y^{pr}(k) & N_z^{pr}(k)\end{bmatrix}^T$ is the predicted values of the external moments.

The accuracy of the prediction is determined by the covariance matrix elements characterizing the second moment of the distribution of the vector x;

$$E\left[\left(x-x^{pr}\right)^2\right]=D_x^{pr}=\left(\frac{\partial\hat{f}}{\partial(\Delta x)^T}\right)P_{st}\left(\frac{\partial\hat{f}}{\partial(\Delta x)^T}\right)^T \tag{22}$$

where P_{st} is the covariance matrix of the estimation value errors at the moment that the estimation process of the x vector has been stopped. Substituting $\frac{\partial\hat{f}}{\partial(\Delta x)^T}=F_u$ into the expression (22) we obtain the prediction covariance matrix in the following simple form,

$$E\left[\left(x-x^{pr}\right)^2\right]=D_x^{pr}=F_U P_{st} F_U^{\,T} \tag{23}$$

6. ANALYSIS AND RESULTS

In this section, we give the simulation results obtained. We consider a low Earth orbiting nanosatellite. Satellite has two attitude sensors as magnetometers and sun sensors. For the magnetometer measurements, the sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $300\ nT$ which is suited to 0.008 for the unit vector measurements. The standard deviation for the sun sensor noise is taken as 0.002 for the unit vector measurements. The eclipse period is between 3000-5000 sec and analyzed in section 6.2.

6.1. Simulation Results for the Sun Phase

First part of this study is to compare the traditional and nontraditional attitude and rate estimation approaches based on EKF. For this purpose, the nanosatellite, having magnetometer and sun sensor on-board, has a circular orbit without any eclipse period. The attitude and rate estimations of these filters can be seen in Fig.1-2. As it can be seen from the figures, especially the traditional EKF fails in eclipse period. Therefore, the sun phase is investigated in this section. For this purpose, Root Mean Square (RMS) errors based on the actual values simulated are composed in Table I. Table I indicates that both filters perform reasonably well in the sun-phase, with SVD/EKF having the smallest attitude RMS error. Thus, SVD/EKF method is used for the sun phase. However, two additional methods are compared in the next sub-section in order to find efficient estimations for the satellite's attitude and angular rates during the eclipse.

Table 1. RMS Errors of the Attitude and Angular Rates for the Sun Phase

RMS Error (%)	Traditional Approach (EKF)	Non-Traditional Approach (SVD/EKF)
ϕ	0.069	0.066
θ	0.115	0.070
ψ	0.131	0.124
ω_x	0.022	0.027
ω_y	0.056	0.020
ω_z	0.055	0.016

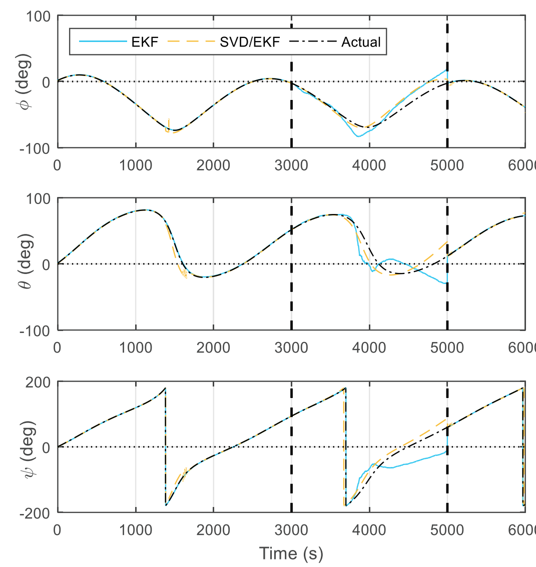

Fig. 1. Attitude estimations using traditional EKF and nontraditional SVD/EKF wrt actual values of the satelllite's attitude angles.

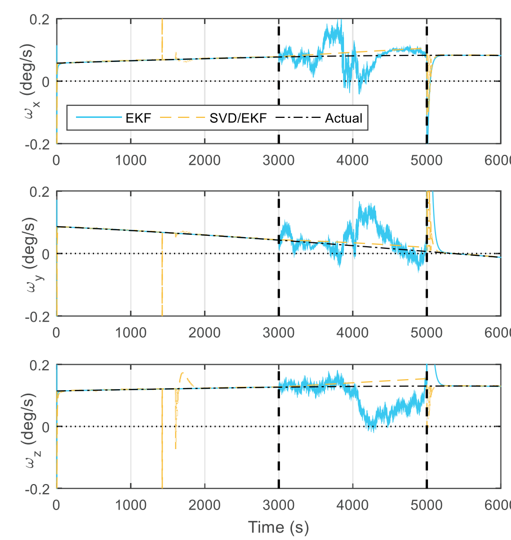

Fig. 2. Angular rate estimations using traditional EKF and nontraditional SVD/EKF wrt actual values of the satelllite's angular velocities.

6.2. Simulation Results for the Eclipse Phase

Any satellite experiencing the eclipse during its orbit needs an eclipse operation mode if it has only sun sensors and magnetometers as attitude sensors. In this case, the sun sensors have zero-output because of the absence of Sun sensor measurements. In this study, two methods are investigated and compared for the eclipse period. First, only magnetometer based traditional EKF described in Section 3 is applied right after the SVD/EKF method is stopped for the eclipse period. It uses the initial conditions as the last estimations of the SVD/EKF before eclipse. For the whole eclipse period, it runs recursively. Then, in the sun-phase, SVD/EKF become active again for being switched from the only magnetometer based traditional EKF. For the second method, a prediction algorithm described in Section 5 is applied. The absolute errors of attitude and rate determination using SVD/EKF, prediction, and EKF based on only magnetometer measurements are seen in Fig.3-4. As it can be seen from the figure, only magnetometer measurements based EKF is not affected from the eclipse since it does not use any information from the Sun. The results of the prediction algorithm running on the satellite for the eclipse period (in loss of Sun sensor measurements**)** are improved by the prediction method then SVD/EKF; however, if the eclipse period were expanded, the results would be deteriorated in time.

Table 2. RMS Errors of the Attitude for the Eclipse Phase

RMS Error (%)	Long-term			Short-term		
	ϕ	θ	ψ	ϕ	θ	ψ
SVD/EKF	0.121	0.133	0.238	0.032	0.104	0.32
Prediction	0.120	0.118	0.215	0.032	0.102	0.31
EKF (only mag.)	0.063	0.074	0.079	0.084	0.052	0.09

A comparison between the methods are performed for short-long term periods of eclipse as 1000 seconds (3000-4000 s) and 2000 seconds (3000-5000 s) along one orbit. RMS errors for the short-long term eclipse phases can be seen in Table II. The attitude determination is achieved using both of the methods in each period of time considered. However, depending on the orbital parameters of the satellite one of the methods can be selected. As our case has a long-term eclipse period, we suggest to use the only magnetometer based traditional EKF in the eclipse phase and SVD/EKF for the sun phase. So, in eclipse, the algorithm is switched into the only magnetometer based traditional EKF method from the base nontraditional attitude estimation approach which is using the covariance knowledge. However, if the eclipse period is not very long, the prediction of attitude parameters based on the mathematical model of the rotational motion of the satellite's center of mass can also be used.

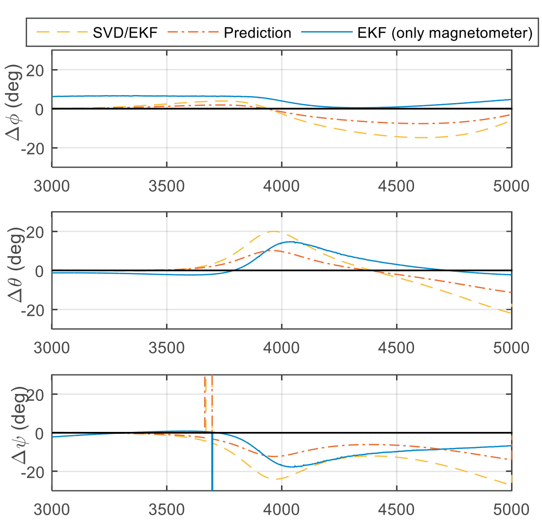

Fig. 3. Absolute errors of the attitude angles by SVD/EKF, prediction, and only magnetometer measurements based EKF in eclipse period.

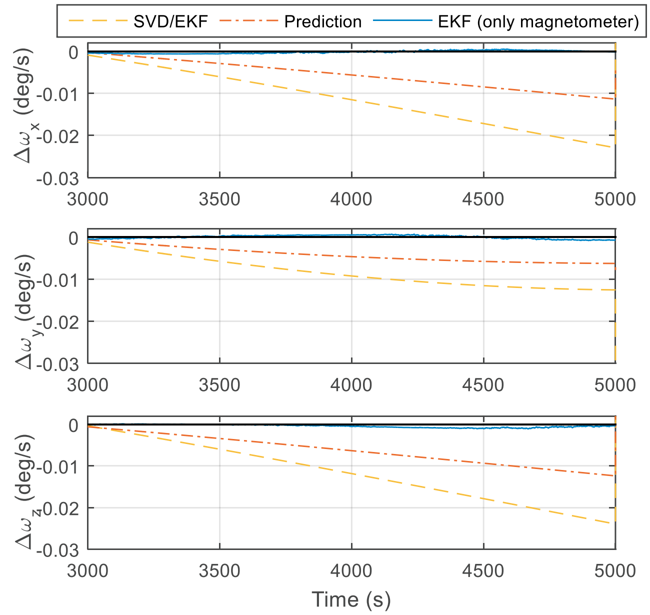

Fig. 4. Absolute errors of the angular velocities by SVD/EKF, prediction, and only magnetometer measurements based EKF in eclipse period.

The suggested switching algorithm based on SVD/EKF and EKF based on only magnetometer methods is presented in Fig. 5.

The errors in eclipse are accumulating while the eclipse period if sun sensor measurements are used in the filter. Therefore, the EKF based on only magnetometer measurements is suggested for this period. However, the accuracy of this method may be improved if an additional sensor is added such as rate gyros.

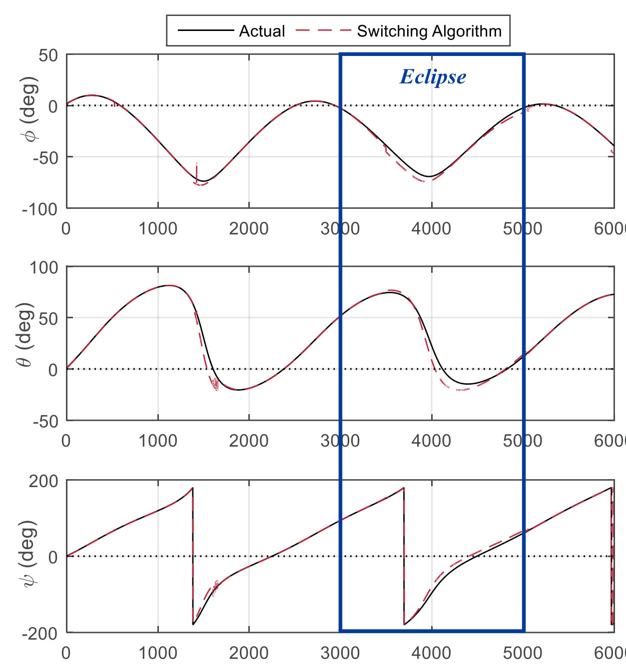

Fig. 5. Attitude estimations using the switching algorithm.

7. CONCLUSIONS

In this study, gyro-free attitude and angular rate estimation methods are introduced with taking into account of the sun and eclipse phases of a nanosatellite. For the sun-phase of the orbit, traditional EKF and nontraditional SVD/EKF based methods are considered in the first stage. Since they both failed in the eclipse period due to loss of Sun sensor measurements, prediction method and only magnetometer based EKF method are considered in the second stage of the analysis. For this purpose, the SVD/EKF with better accuracy than the traditional EKF is stopped and the other method considered for the eclipse is stepped into until the eclipse phase is over.

The drawback of the prediction in eclipse is that, it can be used for only short time intervals. If the errors are accumulated in time while there is no observation from Sun, it is recommended that only magnetometer measurements based EKF can be preferred. By using two methods, a switching algorithm is introduced with the best performance among others.

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