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Inter-generational effect of parental time and its policy implications

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ABSTRACT

Motivated by the empirical fact that parents with more human capital spend more time teaching and taking care of their children, we develop and estimate a theoretical model in which altruistic parents pass their human capital on in two ways: goods investment and time investment. Based on the estimated model, we quantitatively assess how the two types of investment affect wage inequality and inter-generational mobility. Using the model to study the impacts of a public policy that taxes income to finance public schooling, we find significantly different policy effects in our model than in a model where time investment does not respond endogenously to the policy.

1. Introduction

Empirical studies reveal that parental time with children is strongly positively correlated with parents' human capital, proxied by either wage rate or educational attainment. Using the 2003–2006 waves of the American Time Use Survey, Guryan et al. (2008) document a positive wage/education gradient in child care time which holds true for various categories of child care. Similar patterns have been documented in many earlier studies, although in a less comprehensive way. Positive wage gradient of parental time is found in Hill and Stafford (1974), Kimmel and Connelly (2007) and others; while positive education gradient is seen in Leibowitz (1974a, 1974b), DeSimone and Dills (2006), Ramey and Ramey (2010), etc. The positive correlation is also found in countries other than the U.S. (Gauthier et al., 2004; Guryan et al., 2008).

Why would higher wage/education parents spend more time with their children despite the higher opportunity cost? We answer this question based on a simple idea—altruistic parents make both time investment and goods investment to produce their next generation's human capital. If the two types of investment have low substitutability, then higher wage/education parents make more time investment in optimality to complement goods investment.[1]

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[1] Two stylized data facts are consistent with the view that parental time with children is a type of investment. First, in cross-sectional data, parental time with children follows a different pattern than either leisure or home production time—the amount of time allocated to home production and to leisure

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We formalize this idea in a model featuring inter-generational transmission of human capital. We show analytically that the wage gradients of parental investment are positive if time-goods substitutability is low. Further, the gradients are reduced by the public policy that levies a proportional tax on income and provides public schooling.

Ramey and Ramey (2010) provide another channel that leads to the positive wage/education gradient of time investment. Using a model in which parental time is the only input in human capital production, Ramey and Ramey (2010) show that the gradient is positive if the time investment of more educated parents is more efficient. This channel is admitted in an extended version of our model. When human capital production needs both goods and time inputs, we show that whether the gradient is positive also depends on the substitutability between time and goods. When the substitutability is high, parents with more human capital (hence higher efficiency in time investment) should make less time investment and work more to provide more goods investment, unless their advantage in parenting outweighs the advantage on the labor market.[2]

Through time and goods investment, parents partially transmit their human capital to the next generation, thus earnings must exhibit inter-generational persistence. In addition, because richer parents make more investment, parental investment is also a source of long-run wage inequality. In order to assess these inter-generational effects, we estimate the model parameters through the simulated method of moments, then use these parameters to decompose wage inequality and earnings persistence quantitatively.

We also quantitatively analyze the policy effects of public schooling. Since the public policy triggers a reinforcing mechanism among time investment, goods investment and human capital accumulation, it effectively reduces wage inequality and earnings persistence, and increases wage level. In an otherwise similar model that assumes exogenous time investment, the policy effects are much weaker. Given the strong empirical evidence of positive wage gradient, the endogenization of time investment is critical in policy analysis. We further show that if parents with more human capital are more efficient in time investment, the public policy is less effective in reducing inequality and earnings persistence, but more effective in promoting human capital accumulation.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model and presents the analytical results regarding positive wage gradients of goods and time investment. Some analytical results related to policy effects are also obtained. In Section 4, we estimate the parameter values of the model and use them to show how public policy affects resource allocation and wage structure. Quantitative decomposition of wage inequality and earnings persistence are also carried out in this section. Section 5 emphasizes the role played by time investment by comparing results between our model and a model in which time investment is exogenous. Section 6 includes further discussions regarding model setup and model implications. Section 7 concludes.

2. Literature review

Our paper is closely related to Restuccia and Urrutia (2004) which considers a model of inter-generational human capital transmission featuring two types of goods investment: early education, and college education. Many important traits exist both in that paper and ours. Both assume that altruistic parents make all the decisions for children. Individuals are heterogeneous in their own human capital and their children's innate ability which is persistent across generations. In addition, both papers find that innate ability accounts for the majority of inter-generational persistence. The distinctive feature of our model is the role of parental time investment. We compare our model and a variant of their model to show that policy effects are significantly different when parents can respond to the policy with changing time investment.

The paper is also closely related to Glomm and Ravikumar (1992) and Glomm and Ravikumar (2003). Both use dynamic models to explore the interactions among parental investment, inter-generational earnings persistence and long-run wage inequality.

Another strand of related literature empirically studies the inter-generational correlation of earnings and educational attainment. Solon (1992) regresses children's log earnings when adults against parents' log earnings, and obtains the slope coefficient that is around 0.45. Aaronson and Mazumder (2008) also report high inter-generational correlation. Parental income (or parents' educational performance) is also correlated with children's educational outcomes. See Acemoglu and Pischke (2001), Dahl and Lochner (2008) and Tominey (2009), Oreopoulos et al. (2003), Chevalier (2004), Black et al. (2005) and others. The causality between parental income (education) and children's educational attainment has been studied in Blau (1999), Black et al. (2005), and Loken (2010).

Roughly speaking the inter-generational correlation can be caused by "nature effect" and "nurture effect". A large number of studies find that "nature effect" is the key determinant, including Behrman and Taubman (1989), Shea (2000), Sacerdote (2002), Maurin (2002), Plug and Vijverberg (2003), and Huggett et al. (2011). Our structural estimation also finds that "nature effect" accounts for a large fraction of inter-generational earnings persistence.

A key parameter in this paper is the substitutability between time and goods investment for which little direct evidence exists. We use the structure of the model to infer a low substitutability from the data. A few existing empirical studies imply

(*footnote continued*)
falls sharply as income and educational attainment rise (Aguiar and Hurst, 2007; Kimmel and Connelly, 2007; Guryan et al., 2008). Second, parental time exhibits positive effects on children's outcome (Leibowitz, 1974b; Cooksey and Fondell, 1996; Li et al., 2005).

[2] We are grateful to a referee for pointing out the importance of this additional mechanism.

that parental child care can be substituted to some extent. For example, Leibowitz (1974a) shows that child care time is reduced when non-parental child care is present. Baker et al. (2008), Havnes and Mogstad (2009) and Gupta and Simonsen (2010) show that government programs that provide universal day care typically crowd out some child care time from mothers.

On the other hand, there exist a number of empirical studies that are consistent with a low substitutability. Leibowitz (1974a), Bianchi (2000) and Sandberg and Hofferth (2001) show that maternal labor market participation only leads to a small-sized reduction in mother's child care time, and leads to increased child care time of fathers. Using Dutch data, Annemarie Nelen and Fouarge (2011) even find that maternal working hours are positively related with planned activities with children. Also consistent with a low substitutability is the negative impact of maternal labor market participation on child outcome (Ruhm, 2004 and Bernal and Keane, 2011). The literature on the correlation between child outcomes and government provision of universal child care generally favors a low substitutability (Baker et al., 2008; Gupta and Simonsen, 2010; Havnes and Mogstad, 2009).

3. The baseline model and analytical results

In this section we lay out the baseline model and present its properties. The economy is populated by two generations, each has a unit measure of individuals. Each individual lives for two periods. In the first period, she is a child, receiving time and goods investment from the adult to form her own human capital. In the second period, she is an adult, making decisions regarding her time and goods allocation.

3.1. Human capital production

Let h_i be the human capital of the ith adult, e_i and g_i be her time investment and goods investment, respectively, then the human capital of her child is

$$h'_i = z_i A[\alpha e_i^{\sigma} + (1-\alpha) g_i^{\sigma}]^{1/\sigma} \tag{1}$$

where z_i is the child's learning ability, $A>0$ is the aggregate technology level. Since our paper focuses on the role of time investment in determining earnings persistence and wage inequality, we simply view A as a scaling parameter. The model can easily be extended to encompass growth by including a time trend in A. The coefficient $\sigma\in(-\infty, 1]$ governs the substitutability between e_i and g_i. Define $\eta = 1/(1-\sigma)$, then η is the elasticity of substitution between e_i and g_i. We say that e_i and g_i are gross complements if $\eta<1$ ($\sigma<0$), and gross substitutes if $\eta>1$ ($\sigma>0$). The coefficient $\alpha\in(0,1)$ determines the relative share of time investment and goods investment in the production of human capital.

Learning ability z_i is random and exogenous. It evolves according to

$$\ln z'_i = \rho \ln z_i + \epsilon_i \tag{2}$$

where ϵ_i is an i.i.d. random shock drawn from normal distribution with mean $-(1/2)\nu^2/(1+\rho)$ and variance ν^2. It is easy to show that the unconditional mean of learning ability is $E[z_i]=1$.[3]

The formulation in Eq. (1) assumes that the human capital of a child is entirely chosen by the adult. This distinguishes our model from those that assume a child can choose her own human capital stock. For example, Glomm and Ravikumar (1992) assume that a child allocates her time between leisure and human capital production. In this paper we emphasize the role played by parental time in human capital formation and its effect on wage inequality and inter-generational earnings persistence. Therefore, we do not take child's own time allocation decision into account.

3.2. Individual's optimization behavior

An adult is endowed with one unit of time, and allocates it among work, leisure and time investment to maximize lifetime utility.

3.2.1. The optimization problem

The ith adult solves the following optimization problem:

$$\max_{c_i, n_i, e_i, g_i} \ln c_i + \gamma \ln n_i + \beta \ln h'_i$$

subject to Eq. (1) and

$$c_i + g_i = (1-e_i-n_i) w_i \tag{3}$$

$$w_i = h_i \tag{4}$$

where c_i and n_i are consumption and leisure respectively. The relative importance of leisure is governed by γ, and the strength of parental altruism is determined by β. In Eq. (3), $1-e_i-n_i$ is the adult's work hour and w_i is the wage rate which

[3] From the distribution of ϵ_i, $\ln z_i$ is also normally distributed with mean $\mu_{\ln z} = -(1/2)\nu^2/(1-\rho^2)$ and variance $\nu^2_{\ln z} = \nu^2/(1-\rho^2)$. Therefore, z_i follows log-normal distribution with mean $\exp(\mu_{\ln z} + \frac{1}{2}\nu^2_{\ln z}) = 1$.

equals the stock of human capital (h_i). Shocks to learning ability are revealed before the adult makes decisions, hence there is no uncertainty about the outcome of these decisions.

It is assumed that the adult cares only about the child's human capital stock. This modeling strategy, following Glomm and Ravikumar (1992), is very common in the literature of inter-generational transmission of human capital. In our framework, it enables us to derive a closed-form solution to individual's optimization problem and analytical results regarding the effects of public policy.

3.2.2. Optimal decisions

Why do parents with more human capital make more time investment? The question is answered in the solution to the individual's optimization problem. For simplicity we omit subscript i. The following equations present the optimal investment decisions as functions of the adult's human capital stock h[4]:

$$g=\frac{h}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{\sigma/(\sigma-1)}+1\right]} \tag{5}$$

$$e=\frac{\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{\sigma/(\sigma-1)}}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{\sigma/(\sigma-1)}+1\right]} \tag{6}$$

The derivatives of parental investment with respect to human capital stock are given in the following two equations:

$$\frac{dg}{dh}=\frac{\left(\frac{1}{1-\sigma}\right)\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{\sigma/(\sigma-1)}+1}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{\sigma/(\sigma-1)}+1\right]^2} \tag{7}$$

$$\frac{de}{dh}=\frac{-\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{1/(\sigma-1)}}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}h^{\sigma/(\sigma-1)}+1\right]^2} \tag{8}$$

Eqs. (7) and (8) are the wage gradients of parental investment. Both are positive as long as time and goods investment are complements ($\sigma<0$).

Therefore, our model can generate the stylized data fact that parents with more human capital make more time investment. This is delivered by two model assumptions. First, altruistic parents equate their own marginal utility with the marginal product of goods investment, thus richer parents make more goods investment. Second, the two types of investment have a low substitutability, thus richer parents also make more time investment to complement goods investment.

In the literature review, we have discussed empirical studies that are consistent with a low substitutability between time and goods investment. Leibowitz (2003) provides a discussion about why. First, it is difficult to monitor the quality of purchased child care. A hired nanny or babysitter may not spend her time investing in the child's human capital, but enjoying her own time at the expense of the child. Secondly, the psychological and sociological literature emphasizes that parent–child bonding affects child's development, which is further supported by neurobiologists.[5] Thus, even for the activities such as baby sitting, it is not completely safe to say they can be substituted by purchased care.

3.2.3. Evolution of human capital

Plugging the solutions for g and e into Eq. (1), we express the child's human capital h' as a function of her own learning ability (z) and her parents' human capital (h)

$$h'=zD^{(1-\sigma)/\sigma}\left[\left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\sigma)}+h^{\sigma/(1-\sigma)}\right]^{(1-\sigma)/\sigma} \tag{9}$$

where D is a constant defined as $D=(\beta A/(1+\gamma+\beta))^{\sigma/(1-\sigma)}(1-\alpha)^{1/(1-\sigma)}$.

Clearly h' increases with h and z. Therefore in our model wage is persistent inter-generationally for two reasons: inter-generational transmission via parental time and goods investment, and the persistence of learning ability z. Recall that we assumed that z follows an AR(1) process in Eq. (2).

From Eq. (9), h is convergent as long as $D<1$. To see this, it is sufficient to show that $h^*=h^{\sigma/(1-\sigma)}$ is convergent. The law of motion of h^* is

$$h^{*'}=z^{\sigma/(1-\sigma)}D\left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\sigma)}+z^{\sigma/(1-\sigma)}Dh^*$$

Therefore, $h^{*'}$ is a linear function of h^*. Recall that z is a random number with mean one. When $z=1$, as long as $D<1$, the linear function can be represented by a straight line that crosses the 45-degree line in a two-dimensional space, so h^* is

[4] Following Becker (1993), a strand of literature emerged discussing whether parents choose to invest more in the human capital of abler children or not. From Eqs. (5) and (6), it can be seen that our model implicitly assumes that parental investment is independent of children's ability.

[5] See the references in Leibowitz (2003). Also see Belsky (1988).

convergent. When z is random, the function can be represented by perturbations around the straight line, and h^* is still convergent. The convergent h ensures the existence of a stationary equilibrium in which we study inter-generational persistence, wage inequality and policy effects.

3.3. The public policy

It has been advocated that public investment in education reduces the disparity in private investment, hence promotes equal opportunity and social mobility. To access the policy effects in our framework, we introduce into our model a public policy that levies proportional income tax and uses the revenue to provide public schooling. Positing that public schooling can substitute private goods investment to a large extent, we show that the policy reduces wage gradients of both private goods investment and time investment.

3.3.1. Description of policy

The public policy considered here can be fully characterized by the sequence $\{\tau, P_t\}_{t=1}^{\infty}$ where τ is the tax rate and P_t is the total public investment in human capital in period t. Notice that we assume tax rate τ to be time invariant. Given any τ, P_t changes over time until the economy reaches the stationary equilibrium defined below. For simplicity, in the individual's optimization problem below, we drop time subscripts.

3.3.2. Policy effects

In the regime with public schooling, an individual maximizes the same utility function, but subject to different constraints. Specifically, the budget constraint becomes

$$c + g = (1-\tau)(1-e-n)w$$

where w is the before-tax wage rate. With public schooling, human capital stock h evolves according to

$$h' = zA[\alpha e^{\sigma} + (1-\alpha)(g+P)^{\sigma}]^{1/\sigma} \tag{10}$$

where P stands for the public investment in education and $g+P$ is the total goods investment.

Eq. (10) assumes that public investment P is a perfect substitute for private goods investment g, but a poor substitute for time investment e. Perfect substitutability between private and public goods investment is not unusual in the literature.[6] In reality, the substitutability may not be perfect, but should be high. For example, supported by federal and state funding, public schools charge much lower tuition than private ones. Regarding the assumption of a low substitutability between P and e, we emphasize that parental time helps strengthen parent–child bonding which improves learning attitude and learning incentives, as discussed in Section 3.2.2. Thus parental time can hardly be substituted by public investment in education.

When the government makes excessive investment in education, the individual prefers to make negative private investment. We preclude that and assume $g \geq 0$. When the non-negativity constraint of goods investment is not binding, we have the following.

Proposition I. *When time and goods investments are gross complements* ($\sigma < 0$), *the model with public schooling has the following properties*:

1. $\partial g/\partial h > 0$, *and* $\partial e/\partial h > 0$ *as long as the two types of investment have low substitutability* (*a sufficient condition is* $\sigma < -\beta/(1+\gamma)$).
2. *Private goods investment decreases with P, while time investment and human capital accumulation increase with P.*
3. *Both* $\partial e/\partial h$ *and* $\partial g/\partial h$ *decrease with P.*

Proof of the proposition is given in the Appendix. The first property states that wage gradients are still positive in the presence of the policy. Next, the proposition states that public investment in education crowds out private goods investment, but crowds in time investment. Finally, public investment leads to more equalized time and goods investment, which implies lower inter-generational persistence.

Intuitively, the increased time input due to public policy leads to more human capital accumulation, which results in more tax revenue and public investment, which in turn induces increased time investment due to the complementarity. This reinforcing mechanism, as illustrated in Fig. 1, implies that the policy is significantly more effective in increasing wage rate, and in reducing wage inequality and earnings persistence in our model than in a model without endogenous time investment. We return to this point with quantitative analysis in Section 5.2.

When the non-negativity constraint of goods investment is binding, private goods investment is zero. In this case, it is easy to show that wage gradient of time investment is zero. In addition, time investment no longer increases with public investment P. Rather, it decreases with P.[7] This is because, as the government makes excessive public investment, parents find that, relative to

[6] See Fernandez and Rogerson (1998) and Restuccia and Urrutia (2004).
[7] For the proof, see Appendix C.

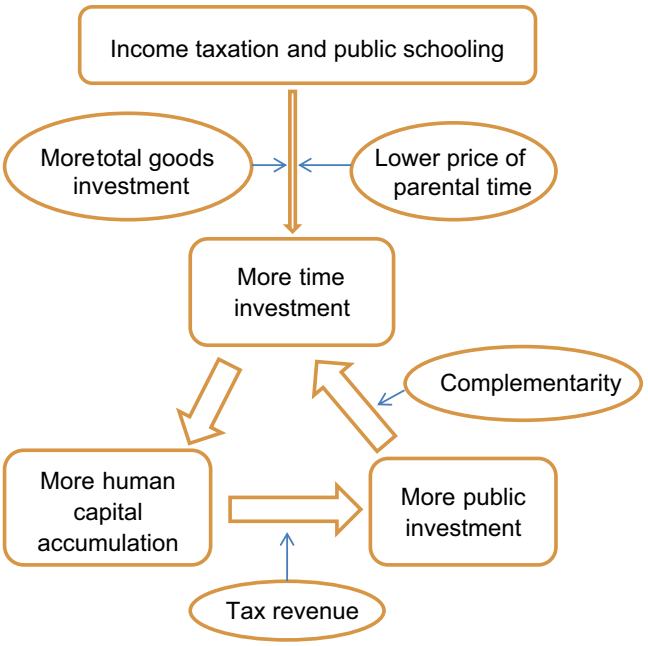

Fig. 1. The reinforcing mechanism. The figure illustrates the reinforcing mechanism among goods investment, time investment and human capital accumulation.

marginal utility of consumption, the marginal utility of investing in the next generation's human capital is too low. Therefore, the optimal strategy for individuals is to reduce time investment, and increase consumption and leisure.

To further clarify the role played by the substitutability between time investment and goods investment, we consider the case of $\sigma > 0$. Proposition II summarizes the results.

Proposition II. *When time and goods investments are gross substitutes* ($\sigma > 0$), *the model with public schooling has the following properties*:

1. $\partial g/\partial h > 0$, *but* $\partial e/\partial h < 0$.
2. *Private goods investment decreases with P, while time investment and human capital accumulation increase with P.*
3. $\partial e/\partial h$ *decreases with P, but* $\partial g/\partial h$ *increases with P.*

Proof of the Proposition II is also given in the Appendix. Notice that the second property in Proposition II is exactly the same as in Proposition I. When $\sigma > 0$, however, the wage gradient of time investment is negative, which is inconsistent with the data. Furthermore, public schooling now leads to more unequal private investment, implying that public schooling is less effective in reducing wage inequality and persistence compared with the case of $\sigma < 0$.

With public investment in education, the explicit solution for $\tilde{h}$ is not obtained, but $\tilde{h}$ satisfies:

$$\tilde{h} = D^{1-\sigma/\sigma}\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)}[(1-\tau)\tilde{h}]^{\sigma/(\sigma-1)}+1\right]^{(1-\sigma)/\sigma}[(1-\tau)\tilde{h}+P] \tag{11}$$

where D is defined the same way as in Eq. (9).

3.4. Distribution and stationary equilibrium

We are interested in the inter-generational earnings persistence and wage inequality implied in our model. In this subsection we discuss the distribution of individuals and provide definitions of persistence, inequality and equilibrium. These definitions prepare us for the quantitative study that follows.

3.4.1. Distribution of individuals

Adult individuals in the economy are heterogeneous in two aspects: own human capital stock (h) and the child's learning ability (z). Individuals are distributed across the state space $H \times Z$ where $H \subset \Re^+$ is the space for adult's human capital stock, and $Z \subset \Re^+$ is the space for child's learning ability.

Let $\lambda_t(h,z)$ be the distribution of individuals across state space $H \times Z$ in period t. We need to know how the distribution evolves over time. The transition of z is given by Eq. (2), and the transition of h is determined by the policy function $h'(h,z)$. Therefore once the policy function is known, we can track the transition of individuals from one generation to another.

To describe the evolution of $\lambda_t(h,z)$ over time formally, let $\mathcal{H}\times\mathcal{Z}$ be a typical subset of $H\times Z$. Define $Q((h,z),\mathcal{H}\times\mathcal{Z})$ as the probability that the next generation of the adults with current state (h,z) transmits to the set $\mathcal{H}\times\mathcal{Z}$. Formally

$$Q((h,z),\mathcal{H}\times\mathcal{Z})=\int_{\mathcal{Z}}\mathcal{I}\{h'(h,z)\in\mathcal{H}\}\,dz \tag{12}$$

where $\mathcal{I}$ is the indicator function. Then the transition of distribution from period t to period $t+1$ is

$$\lambda_{t+1}(\mathcal{H}\times\mathcal{Z})=\int_{H\times Z}Q((h,z),\mathcal{H}\times\mathcal{Z})\,d\lambda_t(h,z) \tag{13}$$

3.4.2. Stationary equilibrium

We have shown the existence of a steady state when the model is stripped of random shocks. Adding shocks back to the model, the economy converges to a stationary distribution $\lambda^*(h,z)$ in which the economy operates around $h=\tilde{h}$ and $z=1$. For any given tax rate τ, a stationary equilibrium is the set of policy functions $h'(h,z)$, $c(h,z)$, $e(h,z)$, $n(h,z)$, $g(h,z)$, the public investment in education P, and a distribution $\lambda^*(h,z)$ such that

1. $h'(h,z)$, $c(h,z)$, $e(h,z)$, $n(h,z)$, $g(h,z)$ solve the adult's optimization problem.
2. Government budget is balanced, i.e., $\int_{H\times Z}\tau(1-e-n)h\,d\lambda^*(h,z)=P$.
3. For any subset $\mathcal{H}\times\mathcal{Z}$, the distribution $\lambda^*(h,z)$ is time invariant. Mathematically

$$\lambda^*(\mathcal{H}\times\mathcal{Z})=\int_{H\times Z}Q((h,z),\mathcal{H}\times\mathcal{Z})\,d\lambda^*(h,z)$$

In the qualitative analysis below, we start from the stationary equilibrium without public policy. After introducing the public policy into the economy, we solve for the evolution of distribution λ_t computationally until the economy reaches a new stationary equilibrium. Using the distributions during the transition period, we are able to compute the transitions of wage, consumption, time allocation, as well as earnings persistence and wage inequality. We measure wage inequality by the coefficient of variation of wage rates. Earnings persistence is defined as the coefficient b_1 in the following standard regression:

$$\ln y'_i=b_0+b_1\ln y_i$$

where y_i and y'_i are the earnings of parents and children, respectively.

The quantitative analysis below involves solving the general equilibrium model numerically. We provide details of the model solution strategy in the Appendix.

4. Quantitative results I: effects of public policy

In this section we quantitatively assess the role played by parental time investment. Further we study how the public policy changes wage level, wage inequality and inter-generational earnings persistence. To do so, we estimate the model parameters via the Simulated Method of Moments.

4.1. Estimation

The model has seven parameters: A, β, α σ, γ, ρ, and ν. The scaling parameter A shifts the levels of human capital stock and other variables. We normalize the steady state human capital stock to one, which implies

$$A=\frac{1+\gamma+\beta}{\beta}\frac{\left[1+\left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\sigma)}\right]^{(\sigma-1)/\sigma}}{(1-\alpha)^{1/\sigma}}$$

Therefore, once we estimate the remainder of the six parameters, A is pinned down.[8] Table 1 recapitulates the definitions of the six parameters to be estimated.

4.1.1. Data moments

We estimate the parameters using the Simulated Method of Moments as formalized in Ingram and Lee (1991). Basically we choose moments that characterize a set of key data features, then search for the parameter values that minimize the distance between data moments and model moments from simulated data. The distance is defined by l^2-norm, weighted by the inverse of the variance of data moments.[9] Table 2 presents the data moments and their sources.

The first three moments are the mean levels of consumption, goods investment and time investment. They are informative about the relative importance of time investment in human capital production and the degree of altruism.

[8] It is easy to show that when A is defined this way, human capital stock is convergent. Based on the estimation results, A is around 11.5.

[9] Ideally we should have used the optimal weighting matrix which is the inverse of the variance-covariance matrix of the data moments. However the off-diagonal components are not available from the data, due to the lack of panel data and the fact that moments are taken from different sources.

The fourth and fifth moments are wage elasticities of parental investment, obtained by regressing the logarithm of goods and time investment on the logarithm of wage rate, instrumented by educational attainment.[10] This moment is very responsive to σ, the substitutability between two types of investment. The sixth moment is informative on ρ, the persistence of learning ability over generations.

The last moment measures the overall wage inequality in the economy. It is very responsive to the size of random shocks to learning ability. The literature on economic inequality uses the variance of logarithm of wage rate as a typical measure of inequality. Since wage rate in the model is much smaller in scale than in the data, we use the coefficient of variation so that the model moment is comparable to that in the data.

4.1.2. Estimation results

Table 3 reports the estimation results. Not surprisingly, σ is negative, which means the elasticity of substitution between goods and time investment is $\eta = 1/(1-\sigma) = 1/(1-(-0.909)) \approx 0.524$. The estimate of inter-generational altruism, β, is 0.246. If we consider one generation to be 20 years, then this is equivalent to an annual discount factor of 0.932. Regarding the two parameters for the unobservable stochastic learning ability process, we find that $\rho = 0.315$ and $\nu = 0.522$. Overall our model delivers the data facts very well, except that the wage elasticities of time and goods investment are slightly lower in the model than in the data.

4.2. Inter-generational persistence and wage inequality

Earnings persistence and wage inequality have two sources: exogenous learning ability and parental investment, which correspond to "nature" and "nurture" effects in the literature respectively. In order to show the role played by the latter, we decompose earnings persistence and wage inequality quantitatively. First we calculate them in the case in which every individual makes the steady state level time and goods investment. Next, we keep time investment fixed at the steady state level, but allow goods investment to be endogenously chosen. Finally, we allow both types of investment to be endogenous. Different results from these cases, reported in Table 4, reveal the roles played by time and goods investment.

Table 4 shows that parental investment accounts for a relatively small portion of the wage inequality and earnings persistence. Fixing e and g at the steady state levels, wage inequality is 0.590, while it is 0.663 when both e and g are endogenous. Loosely speaking, parental investment, the so-called "nurture effect", accounts for 11% of overall wage inequality. It accounts for 36.4% of earnings persistence. Within "nurture effect", both time investment and goods investment are important contributors.[11]

Columns (2) and (5) of Table 4 report wage inequality and earnings persistence in the new stationary equilibrium to which the economy reaches gradually after the public policy is implemented.[12] Here tax rate is assumed to be 5%, and $\tilde{g}$ is defined as public goods investment plus steady state private goods investment. As we allow goods investment to be endogenous and keep time investment fixed, both inequality and earnings persistence increase, but to a much less extent compared with the no policy case. Columns (3) and (6) show the policy effects. Allowing both time and goods investment to be endogenous, the public policy reduces inequality by 6.6% and persistence by 14.9%. Clearly, our model with endogenous time investment predicts a much stronger policy effect. In Section 5.2, we explicitly compare policy effect of our model with a traditional model that abstracts from endogenous time investment.

We further decompose the above policy effects by conducting two experiments. Results are reported in Table 5. First, we impose 5% tax, but assume no public investment in education. This leads to slightly higher inequality and persistence, because income tax strengthens the inequality in time investment. In addition, the "tax only" policy leads to slightly lower time and goods investment. Next, we consider the "public investment only" policy that keeps public investment but assumes away income taxation. The results reveal that much of the policy effects in the previous discussion are due to public investment, rather than income taxation.

5. Quantitative results II: the role of time investment

The reinforcing mechanism illustrated in Fig. 1 exists in our model, but does not exist in the traditional-type model in which parental time investment is exogenous. Therefore in our model, the policy is much more effective in promoting human capital accumulation, and in reducing inter-generational earnings persistence and wage inequality. We quantitatively compare the policy effects between our model and a version of the traditional model in which time investment is exogenous.

[10] We use education attainment to instrument wage rate because we want to capture the human capital element in wage rate. In particular, this filters out shocks to wage rate which are not directly related to human capital.

[11] As pointed out by Cunha and Heckman (2007), "nurture effect" and "nature effect" on inter-generational persistence are nonlinear, and they interact with each other. Our production function of human capital is clearly non-linear.

[12] For the reader interested in the dynamics of the economic transition, we plot transition paths of all the relevant variables in Appendix D.

Table 1
Model parameters.

Symbol	Definition
β	Degree of altruism toward children
α	Relative importance of time investment
σ	Substitutability between time and goods investment
γ	Relative importance of leisure in preference
ρ	Inter-generational persistence of learning ability
ν	Standard deviation of shocks to learning ability

The table summarizes the symbols and definitions of parameters used in the model.

Table 2
Data moments.

	Value	Standard error	Source
Average consumption	0.469[a]	0.003	CEX 2003
Average goods investment	0.025[a]	0.016	CEX 2003
Average time investment	0.086[b]	0.002	ATUS 2003
Wage elasticity of goods investment	1.067	0.371	CEX 2003
Wage elasticity of time investment	0.206	0.077	ATUS 2003
Inter-generational earnings correlation	0.449	0.095	Solon (1992)
Coefficient of variation of wage	0.663	0.021	ATUS 2003

The table reports the data moments that we match in the structural estimation.
[a] Relative to wage/(1−saving rate).
[b] A fraction of total time available.

Table 3
Estimation results.

β	σ	α	γ	ρ	ν
0.246	−0.909	0.893	0.883	0.315	0.522
(0.005)	(0.006)	(0.041)	(0.049)	(0.053)	0.017

Moment	Goods invest.	Time invest.	Consump.	$\frac{\partial g}{\partial h}\times\frac{h}{g}$	$\frac{\partial e}{\partial h}\times\frac{h}{e}$	Earnings persist.	Wage ineq.
Model	0.026	0.086	0.469	0.744	0.188	0.449	0.663
Data	0.025	0.086	0.469	1.067	0.207	0.449	0.663

The table reports estimated parameter values and standard errors. Moments from simulated data are reported together with those from real data.

5.1. The traditional model

We consider a traditional type model that is identical to our baseline model, except that the human capital production function has the following form:

$$h' = zA[B + (1-\alpha)(g+P)^{\sigma}]^{1/\sigma}$$

Here, we have replaced αe^{σ} in our baseline model with a constant B. In the quantitative exercise below, B is calculated based on αe^{σ}, using the estimated α, σ and the average time investment in stationary equilibrium before public policy. This human capital production function can be seen as a variant of the one that Restuccia and Urrutia (2004) use, thus we refer to the traditional model as RU model.

5.2. Comparing policy effects

To compare the policy effects, for each model, we compute the stationary equilibria in two regimes: with and without public policy, then compute the percentage changes of variables due to public policy. Differences in results between the two models reflect the role played by time investment. Columns (1)–(4) of Table 6 present the results.

Both models predict a decrease in work hours, which is due to the reduced after-tax wage rate, but the decrease is more significant in our model where individuals have the option to make more time investment. Both models predict increased leisure, and the increase is again much more significant in our model. Similar changes of time use patterns are found under different tax rates.

Table 4
Wage inequality and earnings persistence.

	Wage inequality			Earnings persistence		
	Without policy (1)	With policy (2)	Policy effect (%) (3)	Without policy (4)	With policy (5)	Policy effect (%) (6)
$e=\tilde{e}$, $g=\tilde{g}$	0.590	0.590		0.286	0.286	
$e=\tilde{e}$, $g=g(h,z)$	0.631	0.606	−4.1	0.398	0.352	−11.6
$e=e(h,z)$, $g=g(h,z)$	0.663	0.619	−6.6	0.449	0.382	−14.9

The table reports the decomposition of wage inequality and earnings persistence. The "with policy" case corresponds to an economy where a 5% income tax is imposed with the revenue invested in public schooling. $\tilde{e}$ and $\tilde{g}$ are time and goods investment in the steady state.

Table 5
Further decomposition of policy effects.

	Time investment (%) (1)	Goods investment (%) (2)	Wage rate (%) (3)	Wage inequality (%) (4)	Earnings persistence (%) (5)
Tax+public inv.	6.1	−84.6	8.4	−6.6	−14.9
Tax only	−0.8	−4.2	−1.6	0.4	0.9
Public inv. only	6.4	−82.3	9.1	−6.3	−14.0

The table reports percentage changes due to different policies. The "Tax only" case imposes 5% income tax rate, but tax revenue is not invested in public schooling. The "Public investment only" case keeps public investment from the baseline model unchanged, but assumes away income taxation.

Private goods investment demonstrates huge drops in both models. The drop is larger in the traditional model. This is because the marginal return on goods investment is relatively lower due to the inability of individuals to increase time investment. Total goods investment is increased less in the traditional model.

Regarding the wage structure, the public policy is significantly more effective in increasing wage rate, and in reducing wage inequality and earnings persistence in our model. With $\tau=5\%$, average wage rate is increased by 8.43%, as opposed to 3.88% in the traditional model. Wage inequality and earnings persistence are decreased by 6.6% and 14.9% respectively in our model, but only 3.7% and 11.0% in the traditional model. Similar observations have already been made in Table 4 where the relative importance of both goods investment and time investment is demonstrated.[13]

Columns (5)–(8) report the results from an alternative set of parameters given in Table 7. These are the parameters that minimize the distance between data moments and moments from the traditional model. Clearly, our results are robust to different parameterizations.

6. Further discussions

In this section we offer further discussions regarding model implication and alternative ways to set up the model. Specifically, we study (i) the extended model in which parents with more human capital are more efficient in parenting; (ii) other forms of parental altruism; and (iii) cross country comparison of parental time and public spending on education.

6.1. Higher efficiency of time investment by more educated parents

Ramey and Ramey (2010) point out that more educated parents should be more efficient in time investment, hence wage/education gradient of parental time should be positive. We extend our baseline model to admit this potentially important channel.[14]

6.1.1. Human capital production function and optimal time investment

Let the human capital production function be

$$h'_i = z_i A[\alpha(h_i^{\delta}e_i)^{\sigma} + (1-\alpha)g_i^{\sigma}]^{1/\sigma} \tag{14}$$

The term h_i^{δ} in Eq. (14) allows for the channel in Ramey and Ramey (2010). Since goods investment is absent in their framework, our human capital production function is essentially the same as in Ramey and Ramey (2010) if $\alpha=1$.

[13] Policy effect in the second row of Table 4 is slightly different from that in column (2) of Table 6. This is because in this section time investment is fixed at the average level in the stationary equilibrium, while it was fixed at the steady state level previously.

[14] We are grateful to a referee for suggestions that led to the development of this subsection.

Table 6
Effects of public policy (percentage change).

	Baseline parameters				Alternative parameters			
	$\tau = 5\%$		$\tau = 2\%$		$\tau = 5\%$		$\tau = 2\%$	
	Our (1)	RU (2)	Our (3)	RU (4)	Our (5)	RU (6)	Our (7)	RU (8)
Δ Work hours	−3.10	−0.22	−1.44	−0.37	−3.22	−0.18	−1.58	−0.43
Δ Leisure	2.46	0.26	1.41	0.45	2.21	0.18	1.32	0.43
Δ Time invest.	6.10	0.00	1.56	0.00	6.31	0.00	1.64	0.00
Δ Parental goods invest.	−84.6	−86.5	−38.2	−39.3	−83.3	−85.4	−38.5	−39.4
Δ Total goods invest.	18.1	14.8	0.7	−0.3	21.0	17.6	0.7	−0.1
Δ Wage rate	8.43	3.88	1.16	0.01	9.29	4.66	1.15	0.02
Δ Wage inequality	−6.60	−3.71	−0.94	−0.13	−7.66	−4.68	−1.12	−0.22
Δ Persistence	−14.9	−11.0	−0.6	1.3	−14.1	−10.7	−0.6	1.1

The table reports policy effects in our model as opposed to the traditional model (RU) in which parental time investment is exogenous. Columns (1)–(4) show the comparison based on baseline parameter values (Table 3). Columns (5)–(8) are based on parameter values that minimize the distance between data moments and moments from the traditional model (Table 7).

Table 7
Calibration based on the traditional model.

β	α	σ	γ	ρ	ν
0.230	0.854	−0.697	0.883	0.315	0.522

The table reports parameter values that minimize the distance between data moments and simulated moments from the traditional model.

When $\delta = 0$, we are back in the baseline model. With non-zero δ, it is straightforward to show the following:

$$\frac{\partial g_t}{\partial h_t} = \frac{\left(\frac{1-\sigma\delta}{1-\sigma}\right)\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)} h_t^{(1-\delta)\sigma/(\sigma-1)} + 1}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)} h_t^{(1-\delta)\sigma/(\sigma-1)} + 1\right]^2} \tag{15}$$

$$\frac{\partial e_t}{\partial h_t} = \frac{-\left(\frac{(1-\delta)\sigma}{1-\sigma}\right)\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)} h_t^{(1-\delta)\sigma/(\sigma-1)}}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{1/(\sigma-1)} h_t^{(1-\delta)\sigma/(\sigma-1)}+1\right]^2} \tag{16}$$

Given these equations, there exist two cases in which wage/education gradients of goods and time investment are positive.

Case (i), $\delta < 1$ and $\sigma < 0$. In this case, for parents with more human capital, the efficiency of time investment does not outweigh their advantage on the labor market. Complementarity between two types of investment is still needed for the wage/education gradients to be positive.

Case (ii), $\delta > 1$ and $\sigma > 0$. In this case, parents with higher human capital are much more efficient in time investment. $\partial e_t/\partial h_t$ will be positive if time and goods investment are substitutes. In addition, from Eq. (15), if δ is large enough, goods investment will decrease with human capital. This is because for parents with more human capital, the advantage in parenting outweighs the advantage on the labor market, and they substitute goods investment for time investment.

Since more educated women are more likely to be in the labor force, it is likely that education has the non-neutral effect of raising the productivity of labor market time more than that of time spent in parenting. Therefore, we focus our discussion on case (i). When the same public policy is introduced into our model, we find that all the results in Propositions I and II hold. The exception is that the sufficient condition for positive wage gradient of time investment becomes $\sigma < -\beta(1-\delta\sigma)/(1+\gamma)(1-\delta)$.[15]

6.1.2. Re-estimation of the model

To conduct a quantitative study based on the extended model, we re-estimate the model with δ as an additional parameter. The estimation yields $\delta = 0.045$. However, the standard errors of σ, ρ and δ are large. The model fails to precisely identify time-goods substitutability (σ), persistence in learning ability (ρ) and higher productivity of more able parents (δ), because the product of δ and σ enters the exponential term in the production of human capital. We proceed by fixing δ at different values and estimating the rest of the parameters. Generally we find the distance between model and data

[15] Proof of the proposition with δ in human capital production function is available upon request.

Table 8
Estimation results with fixed δ.

	β	σ	α	γ	ρ	ν
$\delta=0.045$	0.246	**−0.915**	0.894	0.884	**0.321**	0.519
	(0.01)	(0.05)	(0.01)	(0.06)	(0.08)	(0.02)
$\delta=0.3$	0.246	**−2.159**	0.971	0.884	**0.066**	0.521
	(0.01)	(0.21)	(0.01)	(0.06)	(0.08)	(0.02)

	Goods invest.	Time invest.	Consump.	$\frac{\partial g}{\partial h}\times\frac{h}{g}$	$\frac{\partial e}{\partial h}\times\frac{h}{e}$	Earnings persist.	Wage ineq.
$\delta=0.045$	0.025	0.086	0.469	1.067	0.207	0.449	0.663
$\delta=0.3$	0.025	0.086	0.469	0.741	0.189	0.449	0.663

The table reports estimation results for the extended model for two cases, $\delta=0.045$ and $\delta=0.3$. Implied model moments are also reported.

moments increases slightly with δ, supporting a low δ.[16] Table 8 reports the results with δ fixed at 0.045 and 0.3. When δ is higher, the estimated persistence in learning ability ρ is much lower. Substitutability between goods and time is also lower, which enables the model to match the positive wage gradient of goods investment in the data.

6.1.3. Policy effects in the extended model

Table 9 reports the policy effects given different values of δ. A higher δ means more of the inter-generational effect is driven by time investment. Compared with the baseline case, the public policy induces even more parental investment, hence is less effective in reducing the inter-generational effect. The table also shows that if time investment is fixed ($e=\tilde{e}$), then policy effects increase with δ.

6.2. Other forms of parental altruism

For transparency and tractability, this paper assumes a simple form of parental altruism—parents care only about the wage rate of their children. In this subsection we discuss other forms of altruism.[17]

6.2.1. Financial transfer

We extend our model to admit financial transfer from parents to children. Let b_i be the transfer from the ith parents who maximize

$$\max_{c_i,n_i,e_i,g_i,b_i} \ln c_i+\gamma\ln n_i+\beta\ln h'_i+\phi\ln b_i$$

subject to

$$c_i+g_i+b_i=(1-e_i-n_i)w_i \tag{17}$$

Let g^* and e^* be the optimal goods and time investment in the baseline model given $\{h_i,z_i\}$, the optimal goods and time allocation in this extended model is

$$g=\frac{1+\gamma+\beta}{1+\gamma+\beta+\phi}g^*$$

and

$$e=\frac{1+\gamma+\beta}{1+\gamma+\beta+\phi}e^*$$

Therefore compared with the baseline model, optimal time and goods investment are reduced by a fixed proportion. As a result, the analytical properties regarding time allocation and policy effects derived in the previous sections still hold. Since parents allocate relatively less time and goods to human capital production compared with the baseline case, the extended model must attribute a larger part of observed intergenerational persistence to the "nature effect", i.e., the persistence in learning ability. Hence quantitatively public policy would be less effective.

[16] The existing literature provides some indirect evidence in support of a positive δ. For example, more educated parents are less likely to use non-parental child care (Leibowitz, 1974a). Subsidized non-parental care tends to have positive impact on outcomes for disadvantaged children whose parents on average have lower income and education attainment, but negative impact for children from more educated families. See Blau and Currie (2006) for a comprehensive survey.

[17] We are grateful to a referee for suggestions which led to the development of this subsection.

Table 9
Wage inequality and earnings persistence from different δ.

	Wage inequality			Earnings persistence		
	Without policy (1)	With policy (2)	Policy effect (%) (3)	Without policy (4)	With policy (5)	Policy effect (%) (6)
$\delta=0.045$						
$e=\tilde{e}$, $g=\tilde{g}$	0.589	0.589		0.283	0.283	
$e=\tilde{e}$, $g=g(h,z)$	0.631	0.605	−4.1	0.398	0.350	−11.9
$e=e(h,z)$, $g=g(h,z)$	0.663	0.623	−6.1	0.449	0.390	−13.4
$\delta=0.3$						
$e=\tilde{e}$, $g=\tilde{g}$	0.585	0.585		0.246	0.246	
$e=\tilde{e}$, $g=g(h,z)$	0.628	0.600	−4.4	0.388	0.327	−15.7
$e=e(h,z)$, $g=g(h,z)$	0.663	0.665	0.4	0.449	0.448	−0.3

The table reports the decomposition of persistence and inequality for two cases, $\delta=0.045$ and $\delta=0.3$. Overall policy effects are weaker when δ is larger.

6.2.2. *Infinite horizon*

Another form of parental altruism is to assume that parents care about the value of their children, which include children's own utility and the value of grandchildren. The problem essentially becomes an infinite horizon one with the following objective function for parents:

$$\max_{c,n,e,g} E\sum_{t=1}^{\infty}\beta^t u(c_t,n_t)$$

where $u(c_t,n_t)=\ln c_t+\gamma\ln n_t$ is the utility of the tth generation, and E is the expectation operator, taken with respect to the learning ability of future generations.

In this setup, the time-goods investment ratio is the same as in the baseline model which is $e/g=((1-\alpha)/\alpha)^{1/(\sigma-1)}h^{1/(\sigma-1)}$. However, income tax would reduce goods investment to a greater extent. This is because forward-looking parents care about the future generations' consumption and leisure which are directly affected by tax.[18] To show this point, we derive the following first order conditions:

$$(1-\tau)w_t\frac{\partial u(c_t,n_t)}{\partial c_t}=\frac{\partial h_{t+1}}{\partial e_t}\left[(1-\tau)(1-n_{t+1}-e_{t+1})\frac{\partial u(c_{t+1},n_{t+1})}{\partial c_{t+1}}\right]$$
$$\frac{\partial u(c_t,n_t)}{\partial c_t}=\frac{\partial h_{t+1}}{\partial g_t}\left[(1-\tau)(1-n_{t+1}-e_{t+1})\frac{\partial u(c_{t+1},n_{t+1})}{\partial c_{t+1}}\right]$$

It is easy to show that the first order conditions are exactly the same as those in the baseline model, except for the extra term in brackets which is the marginal value of human capital. In the baseline model, the marginal value is 1, because parents care about children's human capital itself. Here, the marginal value is measured by marginal utility, and lowered by tax rate τ. Therefore, compared with the baseline case, parental goods investment is reduced to a greater extent. However, the public investment still stimulates parental time investment due to time-goods complementarity.

6.3. *Parental time and public spending on education – cross country comparison*

In Proposition I, we make two predictions regarding parental time and public spending on education. We check these predictions with cross country data.[19] We use the parental time reported in Guryan et al. (2008) and public spending on education reported by the World Bank. Fig. 2 plots child care time by working women against public spending on education per capita. Consistent with Proposition I, child care time increases with public investment. This relationship also holds for working men. On the other hand, we find a very weak correlation between education gradient of parenting time by working mothers and public spending—correlation coefficient is 0.18 with a standard error of 0.31. In summary we find strong support for the positive correlation between parental time and public investment, but little support for the wage gradient of parental time to decrease with public investment. Admittedly these correlations only serve as a rough check of the model implications. For more strict studies we have to control for many complicating factors, in particular, country fixed effects.

7. Conclusion

This paper is motivated by the data observation that parents of higher human capital spend more time teaching and taking care of their children. We show that this positive gradient is consistent with a model of inter-generational

[18] We thank a referee for pointing this out.
[19] We are grateful to a referee for suggestions which led to the development of this subsection.

transmission of human capital. A key feature of the model is a low substitutability between goods and parental time investment in the production of children's human capital. We estimate the elasticity of substitution and other model parameters via simulated method of moments, then use the model to quantitatively assess the role of time investment.

The model is also used to study a public policy that taxes income and provides public schooling. Since the public policy triggers a reinforcing mechanism among time investment, public investment and human capital accumulation, it is more effective in our model than in a traditional one where endogenous time investment is absent. Given the strong evidence that parents use time investment as a means of inter-generational transmission, we recommend our model as a framework to analyze relevant public policies.

As an extension to our model we allow parents of higher human capital to be more efficient in time investment. Compared with the baseline case, public policy becomes less effective in reducing inequality and inter-generational persistence, but more effective in promoting human capital accumulation.

It is worthwhile to study some other extensions. For example, the model can be enriched by embedding a life-cycle optimization problem in the current overlapping generations framework, which facilitates the study of rich dynamics among consumption, saving and human capital production. This would also allow us to examine how credit constraints impede human capital formation, and how the problem can be relieved by public policies. We leave these extensions and their policy implications for future research.

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Appendix A. Data

In this paper we estimate the model parameters by matching seven key moments in the model to their data counterparts. Except for the moment of inter-generational correlation in earnings which is taken from Solon (1992), all the moments are obtained from the 2003 waves of the American Time Use Survey and the Consumer Expenditure Survey.

The American Time Use Survey is conducted by the U.S. Bureau of Labor Statistics (BLS) and the data are available from the BLS website. Using the time use taxonomy introduced in Aguiar and Hurst (2007), we consider three categories of parental time with children: basic child care, teaching, and playing. Teaching children includes activities such as reading to/with children, talking/listening to children, and helping children with their homework. Clearly these should be regarded as time investment. Much of the time categorized as basic child care and playing with children is beneficial or auxiliary to the development of human capital. For example, “basic child care” includes activities such as picking up children from school, attending children's events; “playing with children” includes playing sports with children, doing arts and crafts with children, etc. Therefore, we take the sum of the three categories as the proxy for parental time investment. We proxy leisure with the time spent on activities related to the following: lawn, garden and houseplants, animals and pets, socializing and relaxing, sports, exercise and recreation, telephone calls, household and personal mail, travel related to these events, eating, sleeping and personal care. This definition of leisure is precisely “leisure measure two” in Aguiar and Hurst (2007).

Consumer Expenditure Survey data are publicly available from the NBER collection. Educational expenditure has three categories in the data. The first one is tuition for college (higher education). The second one is the tuition for Nursery, Elementary, and Secondary Education. This category includes tuition for elementary and high school, payment for private school bus, and other expenses for day care centers and nursery schools. The third category, other educational service, includes tuition for other schools, rental of books and equipment and other school-related expenses, and contributions to educational organizations. We take the sum of the three as the proxy for goods investment in children's human capital. The survey provides no information on whether these expenditures are for children or not, therefore we delete the observations of families whose head of household is younger than 25 or not married or have no children. Consumption is measured by the sum of expenditures on nondurable non-educational goods and services.

We need to calculate educational expenditure normalized by wage rate. We also need to regress educational expenditure on wage rate to compute the response of goods investment with respect to parents' wage rate. Since the data set provides only family level expenditures, the corresponding wage rate should also be at the family level. We take wage rate to be the sum of both spouses' wage divided by the sum of work hours. i.e., $w=(w_{husband}+w_{wife})/(hours_{husband}+hours_{wife})$. Note that for both spouses, CEX provides information about annual wage, weeks worked in the year and hours worked in each week.

In both ATUS and CEX, we delete observations that have one of the following characteristics: (1) no complete information on age, education attainment, number of children, age of children, and wage rate, labor income, marital status, child care time, leisure, work hours; (2) respondents (in ATUS) or heads of household (in CEX) having zero wage; (3) not married; and (4) having no children. In ATUS we also drop respondents who are younger than 21 or older than 65 in order to focus on parents in their working age who allocate time among work, leisure and child care.

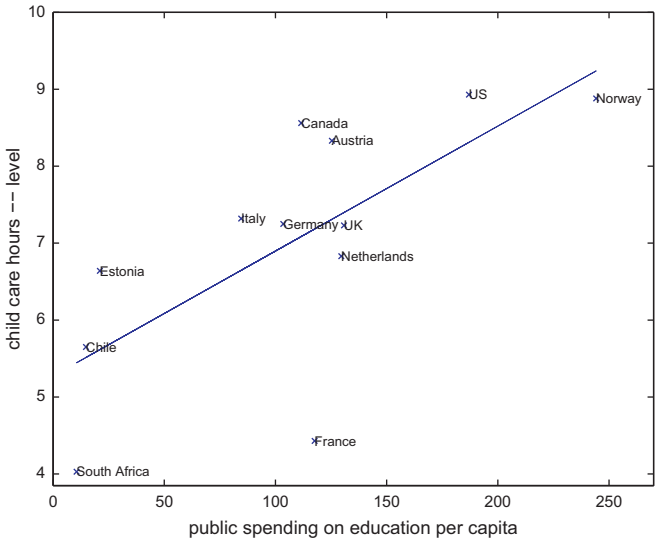

Fig. 2. Public investment and child care time. The figure shows the correlation between public spending on education and child care hours across countries.

Appendix B. Model solution strategy

In the regime with public investment in education, for a given τ, we take the following steps to solve for the stationary equilibrium and the transition paths:

1. Draw random shocks to learning ability for H individuals and T periods, each period corresponding to one generation. Shocks are denoted by $\{\{\epsilon_{i,t}\}_{i=1}^{H}\}_{t=1}^{T}$.
2. Use $\ln z_{i,t} = \rho \ln z_{i,t-1} + \epsilon_{i,t}$ to obtain the paths of stochastic learning abilities for all the individuals. For the ith individual, the path is $\{z_{i,t}\}_{t=1}^{T}$.
3. Guess a sequence of public investment in education $\{P_t\}_{t=1}^{T}$.
4. For each individual, given τ, $\{P_t\}_{t=1}^{T}$, $\{z_{i,t}\}_{t=1}^{T}$, derive the optimal paths of human capital $\{h_{i,t}\}_{t=1}^{T}$, time investment $\{e_{i,t}\}_{t=1}^{T}$ and leisure $\{n_{i,t}\}_{t=1}^{T}$. Whenever analytical solutions are not available (i.e., whenever the non-negativity constraint for goods investment binds), we resort to numerical solutions. Then we calculate tax obligation for the ith individual as $\Phi_{i,t} = (1-e_{i,t}-n_{i,t})\tau h_{i,t}$.
5. Integrate over all the individual to calculate the path of aggregate tax revenue $\{\Phi_t\}_{t=1}^{T}$.
6. Compare tax revenue with public investment in education. Specifically, if the L-infinity norm $|x|_\infty = \max_t |\{\Phi_t\}_{t=1}^{T} - \{P_t\}_{t=1}^{T}|$ is greater than a pre-set tolerance level, find a new guess of $\{P_t\}_{t=1}^{T}$ and repeat steps (3)–(5). To find the new guess, whenever tax revenue is less than the expenditure, P_t is reduced, otherwise P_t is increased.
7. When $|x|_\infty$ is less than the tolerance level, the corresponding $\{P_t\}_{t=1}^{T}$ satisfies the balanced budget condition. Then we solve for the corresponding $\{h_{i,t}C_{i,t}, e_{i,t}, n_{i,t}, I_{i,t},\}_{t=0}^{\infty}$ for each of the individual.
8. Check the distribution of individuals in period $T-1$ and T. If for any subset $\mathcal{H} \times \mathcal{Z}$

$$\lambda_T(\mathcal{H} \times \mathcal{Z}) = \int_{H \times Z} Q((h,z), \mathcal{H} \times \mathcal{Z})\, d\lambda_{T-1}(h,z)$$

then the economy has converged to a new stationary equilibrium. Otherwise, increase T and repeat steps (1)–(7).

Appendix C. Proof of the propositions

Proof of Proposition I. First we derive the following closed-form solutions for g, e, h'. Let $\hat{h} = (1-\tau)h$ be the after-tax wage rate, define the constants $M = (\alpha/(1-\alpha))^{1/(1-\sigma)}$ and $D = (\beta A/(1+\gamma+\beta))^{\sigma/(1-\sigma)}(1-\alpha)^{1/(1-\sigma)}$, then

$$\begin{aligned} g &= \left(\frac{\beta}{1+\gamma+\beta}\right)\left(\frac{\hat{h}+P}{M\hat{h}^{\sigma/(\sigma-1)}+1}\right) - P \\ e &= \left(\frac{\beta M}{1+\gamma+\beta}\right)\left(\frac{\hat{h}+P}{M\hat{h}+\hat{h}^{1/(1-\sigma)}}\right) \end{aligned} \tag{18}$$

and

$$h' = zD(M\hat{h}^{\sigma/(\sigma-1)} + 1)^{(1-\sigma)/\sigma}(\hat{h} + P)$$

When the non-negativity constraint of goods investment is not binding, $g > 0$. From Eq. (18), this implies

$$P < \frac{\beta\hat{h}}{1+\gamma+(1+\gamma+\beta)M\hat{h}^{\sigma/(\sigma-1)}} \tag{19}$$

From these solutions, it is straightforward to show that private investment g decreases with P, time investment e increases with P, and human capital production also increases with P.

The first-order derivatives are

$$\frac{\partial g}{\partial \hat{h}} = \left(\frac{\beta}{1+\gamma+\beta}\right)\left(\frac{1}{1+M\hat{h}^{\sigma/(\sigma-1)}}\right)^2\left(1+\frac{M}{1-\sigma}\hat{h}^{\sigma/(\sigma-1)} - \frac{\sigma MP}{\sigma-1}\hat{h}^{1/(\sigma-1)}\right) \tag{20}$$

and

$$\frac{\partial e}{\partial \hat{h}} = \left(\frac{\beta M}{1+\gamma+\beta}\right)\left(\frac{1}{M\hat{h}+\hat{h}^{1/(1-\sigma)}}\right)^2\left(-\frac{\sigma}{1-\sigma}\hat{h}^{1/(1-\sigma)} - MP - \frac{1}{1-\sigma}\hat{h}^{\sigma/(1-\sigma)}P\right) \tag{21}$$

From Eqs. (20) and (21), it is obvious that both $\partial g/\partial\hat{h}$ and $\partial e/\partial\hat{h}$ decrease with P. Thus $\partial g/\partial h$ and $\partial e/\partial h$ decrease with P. To show $\partial g/\partial h > 0$, we just need to show that the term in the last parenthesis of Eq. (20) is positive. That is

$$\left(1-\frac{M}{1-\sigma}\hat{h}^{\sigma/(\sigma-1)} - \frac{\sigma MP}{\sigma-1}\hat{h}^{1/(\sigma-1)}\right) > 0,$$

which is equivalent to

$$P < \hat{h}\left(-\frac{1}{\sigma} + \frac{\hat{h}^{\sigma/(1-\sigma)}}{-\frac{\sigma}{1-\sigma}M}\right) \tag{22}$$

To show Eq. (22) is implied by Eq. (19), it is sufficient to show

$$\hat{h}\left(-\frac{1}{\sigma} + \frac{\hat{h}^{\sigma/(1-\sigma)}}{-\frac{\sigma}{1-\sigma}M}\right) > \frac{\beta\hat{h}}{1+\gamma+(1+\gamma+\beta)M\hat{h}^{\sigma/(\sigma-1)}}$$

which is equivalent to

$$-\frac{1}{\sigma} + \frac{\hat{h}^{\sigma/(1-\sigma)}}{-\frac{\sigma}{1-\sigma}M} > \frac{\beta\hat{h}^{\sigma/(1-\sigma)}}{(1+\gamma)\hat{h}^{\sigma/(1-\sigma)} + (1+\gamma+\beta)M}$$

which is implied by

$$-\frac{1}{\sigma} + \frac{\hat{h}^{\sigma/(1-\sigma)}}{-\frac{\sigma}{1-\sigma}M} > \frac{\beta\hat{h}^{\sigma/(1-\sigma)}}{(1+\gamma+\beta)M}$$

Since $-1/\sigma > 0$, it is sufficient to show

$$\frac{\beta}{(1+\gamma+\beta)M} - \frac{1}{-\frac{\sigma}{1-\sigma}M} < 0,$$

which is equivalent to $0 < 1+\gamma+\beta-\sigma(1+\gamma)$, clearly true.

To show $\partial e/\partial\hat{h} > 0$, we just need to show the last term in the parenthesis of Eq. (21) is positive, which is equivalent to

$$P < \frac{\hat{h}}{-\frac{1}{\sigma} - \frac{1-\sigma}{\sigma}M\hat{h}^{\sigma/(\sigma-1)}} \tag{23}$$

To show (23) is also implied by Eq. (19), it is sufficient to show

$$\frac{\hat{h}}{-\frac{1}{\sigma} - \frac{1-\sigma}{\sigma}M\hat{h}^{\sigma/(\sigma-1)}} > \frac{\hat{h}}{\frac{1+\gamma}{\beta} + \left(\frac{1+\gamma+\beta}{\beta}\right)M\hat{h}^{\sigma/(\sigma-1)}} \tag{24}$$

Comparing the denominators in inequality (24) reveals that a small σ (large σ in absolute value) guarantees that the inequality holds. The sufficient conditions are $-1/\sigma<(1+\gamma)/\beta$ and $-(1-\sigma)/\sigma<(1+\gamma+\beta)/\beta$, which are equivalent to

$$\sigma<-\frac{\beta}{1+\gamma}$$

This condition is clearly satisfied.

The case of binding non-negativity constraint of goods. If the government makes excessive public investment, the non-negativity constraint for g becomes binding. It is easy to show that, when $g=0$, the optimal time investment satisfies the following equation:

$$\alpha\left(1+\frac{\beta}{1+\gamma}\right)e^{\sigma}-\frac{\alpha\beta}{1+\gamma}e^{\sigma-1}+(1-\alpha)P^{\sigma}=0$$

We cannot solve for e explicitly from the above equation. But since we know $P\geq 0$, it is easy to show that

$$e<\frac{\beta}{1+\gamma+\beta} \tag{25}$$

To show that e decreases with P, define $Q=(1-\alpha)P^{\sigma}$. It is sufficient to show that $\partial Q/\partial e<0$. Since $Q=(\alpha\beta/(1+\gamma))e^{\sigma-1}-(\alpha+\alpha\beta/(1+\gamma))e^{\sigma}$, we have

$$\frac{\partial Q}{\partial e}=\frac{\alpha}{1+\gamma}e^{\sigma-2}[\beta(\sigma-1)-(1+\gamma+\beta)\sigma e]$$

Clearly, $\partial Q/\partial e<0$ when $\sigma<0$ and $e<\beta/(1+\gamma+\beta)$ which is given in (25). □

Proof of Proposition II. When $\sigma>0$, from (21), it is straightforward to see that $\partial e/\partial h$ is negative and decreases with P. From Eq. (20), $\partial g/\partial h$ is positive and increases with P. Given that $\hat{h}=(1-\tau)h$, clearly the properties in Proposition II are true. □

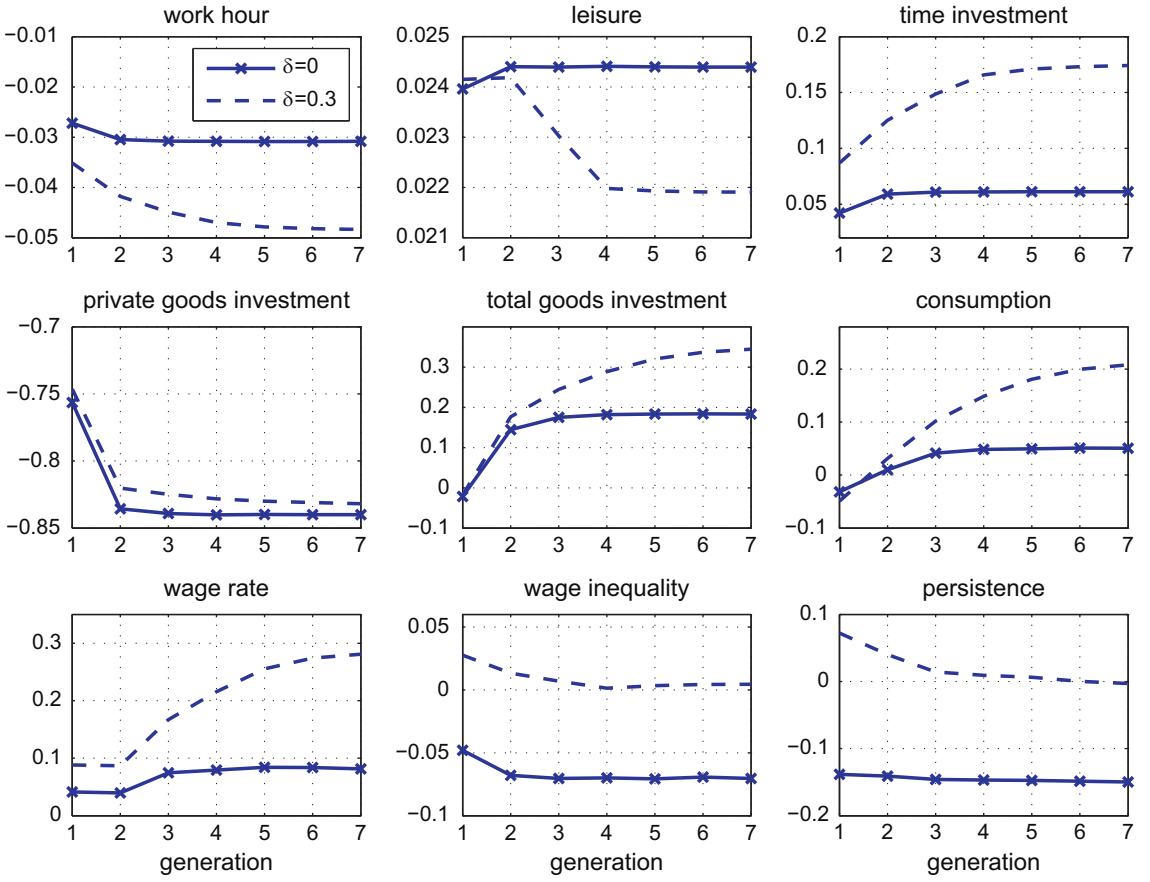

Fig. 3. Transition paths. The figure shows percentage changes due to the implementation of the public policy. The solid lines represent the transition paths in the baseline case; while the dashed lines are for the case in which more educated parents are more efficient in time investment ($\delta=0.3$). Tax rate is 5%.

Appendix D. Transition paths

In the main text we have shown the policy effects by comparing the relevant variables in stationary equilibria. After the implementation of the policy of income taxation and public schooling, the economy transitions from the original stationary equilibrium to a new one. To show a more comprehensive picture of the impact of public policy, we plot the transition paths of the percentage changes of relevant variables in Fig. 3. The solid lines represent the transition paths in the baseline case; while the dashed lines are for the case in which more educated parents are more efficient in time investment ($\delta = 0.3$).

The top panels of the figure plot the percentage changes in time allocation. The policy shifts parents' time from work to leisure and human capital investment. Consistent with the predictions in Proposition I, private goods investment decreases during the transition, but total goods investment and wage rate (human capital) increase, which reflects the reinforcing mechanism among time investment, goods investment and human capital accumulation.

In the baseline model, the policy decreases wage inequality and intergenerational persistence significantly. However, in case of $\delta = 0.3$, the policy fails to reduce inequality and persistence. The reason is that a high δ implies much higher efficiency of time investment by more educated parents. This leads to more disparity in time investment when income is taxed and public schooling is provided. Therefore, higher δ implies that public policy is less effective in reducing inequality and inter-generational persistence but more effective in promoting human capital accumulation.

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