

Exploring fully heavy scalar tetraquarks $Q Q \bar{Q} \bar{Q}$ S.S. Agaev^a, K. Azizi^{b,c,*}, B. Barsbay^d, H. Sundu^e^a Institute for Physical Problems, Baku State University, Az-1148 Baku, Azerbaijan^b Department of Physics, University of Tehran, North Karegar Avenue, Tehran 14395-547, Iran^c Department of Physics, Doğuş University, Dudullu-Ümraniye, 34775 Istanbul, Türkiye^d Division of Optometry, School of Medical Services and Techniques, Doğuş University, 34775 Istanbul, Türkiye^e Department of Physics Engineering, Istanbul Medeniyet University, 34700 Istanbul, Türkiye

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ABSTRACT

The masses, current couplings and widths of the fully heavy scalar tetraquarks $X_{4Q} = Q Q \bar{Q} \bar{Q}$, $Q = c, b$ are calculated by modeling them as four-quark systems composed of axial-vector diquark and antidiquark. The masses $m^{(\prime)}$ and couplings $f^{(\prime)}$ of these tetraquarks are computed in the context of the QCD sum rule method by taking into account a nonperturbative term proportional to the gluon condensate $\langle \alpha_s G^2/\pi \rangle$. Results $m = (6570 \pm 55)$ MeV and $m' = (18540 \pm 50)$ MeV are used to fix kinematically allowed hidden-flavor decay channels of these states. It turns out that, the processes $X_{4c} \rightarrow J/\psi J/\psi$, $X_{4c} \rightarrow \eta_c \eta_c$, and $X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$ are possible decay modes of X_{4c} . The partial widths of these channels are evaluated by means of the couplings g_i , $i = 1, 2, 3$ which describe strong interactions of tetraquark X_{4c} and mesons at relevant vertices. The couplings g_i are extracted from the QCD three-point sum rules by extrapolating corresponding form factors $g_i(Q^2)$ to the mass-shell of a final meson. The mass of the scalar tetraquark X_{4b} is below the $\eta_b \eta_b$ and $\Upsilon(1S)\Upsilon(1S)$ thresholds, therefore it does not fall apart to these bottomonia, but transforms to conventional particles through other mechanisms. Comparing $m = (6570 \pm 55)$ MeV and $\Gamma_{4c} = (110 \pm 21)$ MeV with parameters of structures observed by the LHCb, ATLAS and CMS collaborations, we interpret X_{4c} as the resonance $X(6600)$ reported by CMS. Comparisons are made with other theoretical predictions.

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1. Introduction

Conventional hadron spectroscopy encompasses variety of quark-antiquark mesons and three-quark (antiquark) baryons with different contents and spin-parities. But existence of multiquark particles composed of more than three valence partons is not forbidden by any physical theory or model. Features of such exotic states became object of theoretical studies just after invention of quark-parton model and non-abelian field theory of strong interactions.

Quantitative investigations of multiquark hadrons started from analyses performed by Jaffe in Refs. [1,2] using MIT quark-bag model. In Ref. [1] he made an assumption about four-quark $q^2 \bar{q}^2$ nature of light mesons from the lowest scalar nonet to explain the mass hierarchy of these particles. Another intriguing result is connected with a state composed of six light quarks $S = uudss$ [2]. This double-strange multiquark compound would be stable against strong decays provided such particle really exists. Then hexaquark S may transform to ordinary hadrons only through weak processes and, as a result, have mean lifetime $\tau \approx 10^{-10}$ s, which is considerably longer than that of conventional mesons.

Stability against strong and/or electromagnetic decays is an important question of exotic mesons' physics: Stable four-quark particles (tetraquarks) with long mean lifetime may be discovered in various hadronic processes relatively easily. Therefore, theoretical investigations of such tetraquarks were and remain on agenda of high energy physics. Compounds containing heavy $Q Q$ diquarks ($Q = c$ or b) and light antidiquarks are real candidates to stable exotic mesons. A group of hypothetical particles $Q Q \bar{Q}^{(\prime)} \bar{Q}^{(\prime)}$ and $Q Q \bar{q} \bar{q}$ were explored already in Refs. [3–5], in which it was shown that exotic mesons built of only heavy quarks are unstable particles. But states with content $Q Q \bar{q} \bar{q}$

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may form stable structures if the ratio m_Q/m_q is large. Conclusions about stable nature of the isoscalar axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}^-$ was also made in Ref. [6], whereas four-quark mesons with heavy diquarks bc and cc may be either stable or unstable particles.

More detailed analysis of fully heavy four-quark mesons $X_{4c} = cc\bar{c}\bar{c}$, $X_{2bc} = bc\bar{b}\bar{c}$ and $X_{4b} = bb\bar{b}\bar{b}$ was performed in Refs. [7–12], in which different features of these particles were explored by means of numerous methods and schemes. For instance, in Ref. [7] masses of fully heavy tetraquarks were found by solving nonrelativistic Schrodinger equation. In accordance with this article scalar and axial-vector tetraquarks X_{4c} , X_{2bc} are under the di- J/ψ and $J/\psi\Upsilon(1S)$ thresholds, and only tensor particles can be seen in di- J/ψ and $J/\psi\Upsilon(1S)$ invariant mass distributions. At the same time, all fully beauty exotic mesons X_{4b} reside below $\Upsilon(1S)\Upsilon(1S)$ threshold, and cannot be observed in this mass distribution. Masses of scalar tetraquarks X_{4c} and X_{4b} were estimated also in Ref. [8]. Results obtained there $m(X_{4c}) = (6192 \pm 25)$ MeV and $m(X_{4b}) = (18826 \pm 25)$ MeV allowed the authors to study decay channels and productions of these particles. Because $m(X_{4c})$ is below di- J/ψ but above $\eta_c\eta_c$ thresholds, X_{4c} does not decay to $J/\psi J/\psi$ mesons, while a process $X_{4c} \rightarrow \eta_c\eta_c$ is the kinematically allowed mode. Similarly, X_{4b} cannot decay to a pair of mesons $\Upsilon(1S)\Upsilon(1S)$, whereas $X_{4b} \rightarrow \eta_b\eta_b$ is its possible channel. Interesting predictions about particles X_{4c} and X_{4b} were made in Ref. [10], in which masses of states $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with different spin-parities were calculated by applying the sum rule method. It was demonstrated that masses of the scalar $J^P = 0^{++}$ tetraquarks X_{4c} and X_{4b} , except ones composed of pseudoscalar components, vary inside limits 6.44 – 6.59 GeV and 18.45 – 18.59 GeV, respectively. Subsequently, X_{4c} decays to $\eta_c\eta_c$, $J/\psi J/\psi$, and $\eta_c\chi_{c1}(1P)$ meson pairs, whereas X_{4b} is stable against strong decays to hidden-beauty mesons: Presumably a scalar diquark-antidiquark state X_{4b} built of pseudoscalar components can decay to $\eta_b\eta_b$ and $\Upsilon(1S)\Upsilon(1S)$ mesons. In accordance with Ref. [11], the scalar and tensor X_{4c} , have masses 5.99 GeV and 6.09 GeV, and decay to mesons $\eta_c\eta_c$, whereas di- J/ψ channel is forbidden for them.

Experimental studies of two charmonia or bottomonia productions in pp and $p\bar{p}$ collisions provided valuable information on nature and decay channels of fully heavy exotic mesons. Thus, a pair of J/ψ mesons were observed by LHCb, CMS and D0 Collaborations [13–15], respectively. The $J/\psi\Upsilon(1S)$ and $\Upsilon(1S)\Upsilon(1S)$ pairs were detected and investigated by D0 and CMS experiments [16,17]. In the four-quark picture such final states imply production of intermediate states $cc\bar{c}\bar{c}$, $bc\bar{b}\bar{c}$ and $bb\bar{b}\bar{b}$ with their subsequent decays to couples of heavy conventional mesons.

The discovery of the doubly charmed baryon $\Xi_{cc}^{++} = ccu$ by the LHCb Collaboration [18] gave strong impetus to investigations of doubly and fully heavy tetraquarks. Thus, the mass of Ξ_{cc}^{++} was used as an input parameter to estimate the mass of the axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}^-$ [19]. Conclusions about strong-interaction stable nature of the tetraquarks $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ were made on the basis of heavy-quark symmetry as well [20]. Weak decays of stable double-heavy tetraquarks were explored in numerous publications [21–30]. In our articles [23–29], we calculated masses and current couplings of the tetraquarks $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$ and $bc\bar{u}\bar{d}$ with spin-parities $J^P = 0^+$, 1^+ , as well as parameters of the scalar state $bs\bar{u}\bar{d}$. We evaluated full width of these structures by considering their numerous semileptonic and nonleptonic weak decay channels.

The class of fully heavy exotic mesons $Q Q \bar{Q}^{(\prime)} \bar{Q}^{(\prime)}$ were also explored in Refs. [31–35]. Predictions some of these papers [33,34] confirm in a modified form the results discussed above. But there are also publications which contradict to such conclusions. In fact, using lattice simulations the authors of Ref. [31] did not find evidence for tetraquarks X_{4b} with different spin-parities below the lowest thresholds in relevant channels.

Recently, LHCb reported new structures in the di- J/ψ mass distribution extracted from pp data at c.m. energies 7, 8, and 13 TeV [36]. The LHCb observed a threshold enhancement in nonresonant di- J/ψ production from 6.2 to 6.8 GeV with center at 6.49 GeV. A narrow peak at 6.9 GeV, and a resonance around 7.2 GeV were seen as well. The narrow state labeled $X(6900)$ has parameters

$$\begin{aligned} m_1^{\text{LHCb}} &= (6905 \pm 11 \pm 7) \text{ MeV}, \\ \Gamma_1^{\text{LHCb}} &= (80 \pm 19 \pm 33) \text{ MeV}, \end{aligned} \quad (1)$$

when assuming no interference with nonresonant single-parton scattering (NRSPS) continuum, and

$$\begin{aligned} m_2^{\text{LHCb}} &= (6886 \pm 11 \pm 11) \text{ MeV}, \\ \Gamma_2^{\text{LHCb}} &= (168 \pm 33 \pm 69) \text{ MeV}, \end{aligned} \quad (2)$$

by taking into account interference of NRSPS with a threshold enhancement.

This experimental information was detailed and extended by the ATLAS and CMS Collaborations [37,38]. The ATLAS announced three resonances $X(6200)$, $X(6600)$, and $X(6900)$ in the di- J/ψ channel with the parameters

$$\begin{aligned} m_0^{\text{ATL}} &= 6220 \pm 50_{-50}^{+40} \text{ MeV}, \\ \Gamma_0^{\text{ATL}} &= 310 \pm 120_{-80}^{+70} \text{ MeV}, \end{aligned} \quad (3)$$

$$\begin{aligned} m_1^{\text{ATL}} &= 6620 \pm 30_{-10}^{+20} \text{ MeV}, \\ \Gamma_1^{\text{ATL}} &= 310 \pm 90_{-110}^{+60} \text{ MeV}, \end{aligned} \quad (4)$$

and

$$\begin{aligned} m_2^{\text{ATL}} &= 6870 \pm 30_{-10}^{+60} \text{ MeV}, \\ \Gamma_2^{\text{ATL}} &= 120 \pm 40_{-10}^{+30} \text{ MeV}. \end{aligned} \quad (5)$$

The resonance $X(7300)$ with the mass and width

$$\begin{aligned} m_3^{\text{ATL}} &= 7220 \pm 30_{-30}^{+20} \text{ MeV}, \\ \Gamma_3^{\text{ATL}} &= 100_{-70-50}^{+130+60} \text{ MeV}, \end{aligned} \quad (6)$$

was fixed in the $J/\psi\psi'$ channel. The resonances $X(6200)$ and $X(6600)$ belong to an enhancement in a 6.2 – 6.8 GeV region observed by LHCb. It seems reasonable to suppose that LHCb fixed a superposition of these structures. The resonance $X(7300)$ is close to structure at 7.2 GeV reported by LHCb.

Resonances $X(6600)$, $X(6900)$ and $X(7300)$ discovered by CMS and analyzed in the no-interference model have the following masses and widths

$$\begin{aligned} m_1^{\text{CMS}} &= (6552 \pm 10 \pm 12) \text{ MeV}, \\ \Gamma_1^{\text{CMS}} &= (124^{+32}_{-26} \pm 33) \text{ MeV}, \end{aligned} \quad (7)$$

$$\begin{aligned} m_2^{\text{CMS}} &= (6927 \pm 9 \pm 4) \text{ MeV}, \\ \Gamma_2^{\text{CMS}} &= (122^{+24}_{-21} \pm 18) \text{ MeV}, \end{aligned} \quad (8)$$

and

$$\begin{aligned} m_3^{\text{CMS}} &= (7287^{+20}_{-18} \pm 5) \text{ MeV}, \\ \Gamma_3^{\text{CMS}} &= (95^{+59}_{-40} \pm 19) \text{ MeV}, \end{aligned} \quad (9)$$

respectively. Summing up, we can state that there are four resonances in the range 6.2 – 7.3 GeV discovered by different collaborations in the di- J/ψ and $J/\psi\psi'$ mass distributions.

Observations made by LHCb stimulated further detailed studies of fully heavy exotic mesons [39–48]. Needless to say, that all models and technical tools available in high energy physics were activated to explore these problems. Interesting results concerning properties of fully heavy tetraquarks were obtained using the sum rule method in Refs. [39–43]. For example, depending on a type of interpolating current, the mass of the scalar tetraquark $cc\bar{c}\bar{c}$ was found within limits 6.44 – 6.47 GeV [39]. Fully heavy diquark-antidiquark and hadronic molecules were analyzed also in Ref. [42], in which the resonance $X(6900)$ was interpreted as a molecule $\chi_{c0}\chi_{c0}$ or/and a tetraquark built of pseudoscalar ingredients.

The LHCb data were considered in Ref. [46] in the framework of a coupled-channel approach: It was argued that in the di- J/ψ system exists a near-threshold state $X(6200)$ with spin-parities 0^{++} or 2^{++} . Coupled-channel effects may also generate a pole structure identified in Ref. [48] with the resonance $X(6900)$. Analysis performed there allowed the authors also to predict existence of a bound state $X(6200)$, and broad and narrow resonances $X(6680)$ and $X(7200)$, respectively.

Information of the ATLAS and CMS Collaborations considerably clarified status of structures above the di- J/ψ threshold, and generated new interesting assumptions about their nature [49–54]. Indeed, in Ref. [49] the $X(6200)$ was assigned to be the ground-level tetraquark state with $J^{\text{PC}} = 0^{++}$ or 1^{+-} , whereas its first radial excitation was interpreted as $X(6600)$. Using the relativized Godfrey-Isgur diquark model, the authors of Ref. [52] proposed to consider the resonances starting from $X(6200)$ as the $1S$, $1P/2S$, $1D/2P$, and $2D/3P/4S$ tetraquark states. Similar interpretations were suggested in the context of the relativistic quark model as well [50].

As is seen, there are numerous alternatives to describe structures reported by the different collaborations. In present article, we address problems of these new data, and explore the fully charmed tetraquark X_{4c} with $J^{\text{PC}} = 0^{++}$ by calculating its mass, current coupling and width. We model X_{4c} as a diquark-antidiquark structure, and apply the two-point sum rule method to calculate a relevant correlation function including a nonperturbative term $\sim \langle \alpha_s G^2/\pi \rangle$. It turns out, that processes $X_{4c} \rightarrow J/\psi J/\psi$, $X_{4c} \rightarrow \eta_c \eta_c$, and $X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$ are allowed decay modes of X_{4c} . To calculate their partial widths, we make use of the three-point sum rule approach, and compute strong form factors $g_i(q^2)$, $i = 1, 2, 3$ describing interaction of particles at vertices $X_{4c} J/\psi J/\psi$, $X_{4c} \eta_c \eta_c$, and $X_{4c} \eta_c \chi_{c1}(1P)$, respectively. Predictions for strong couplings g_i , obtained after extrapolation of $g_i(q^2)$ to the mass-shell of a final meson, are used to calculate widths of aforementioned decay channels and to estimate full width Γ_{4c} of the tetraquark X_{4c} . Such detailed information places interpretation of X_{4c} on strong bases and leads to reliable conclusions. We evaluate also the mass m' of the state X_{4b} and show that in the axial-axial model X_{4b} is stable against strong decays to two bottomonia. It is worth noting that in the present paper we do not consider other mechanisms of X_{4c} and X_{4b} decays to conventional particles [44,45].

This article is structured in the following way: In Section 2, we calculate masses and current couplings of the tetraquarks X_{4c} and X_{4b} . Strong decay of X_{4c} to $J/\psi J/\psi$ is considered in Sec. 3. Partial widths of the processes $X_{4c} \rightarrow \eta_c \eta_c$ and $X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$ are computed in Sec. 4. Here, we find also the full width Γ_{4c} of the tetraquark X_{4c} . Last section is reserved for discussion of results and concluding notes. Appendix contains the explicit expression of the heavy-quark propagator, and the perturbative part of the spectral density used in mass computations.

2. Spectroscopic parameters of the tetraquarks X_{4c} and X_{4b}

In this section, we calculate the masses $m^{(\prime)}$ and current couplings $f^{(\prime)}$ of the tetraquarks X_{4c} and X_{4b} by means of the QCD two-point sum rule approach [55,56]. It is a powerful nonperturbative method developed to investigate features of conventional mesons and baryons, but can also be applied to study multi-quark hadrons, such as tetraquarks and pentaquarks.

To derive the sum rules necessary for extracting the masses and current couplings of the scalar tetraquarks X_{4c} and X_{4b} , we begin from analysis of the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ J(x) J^\dagger(0) \} | 0 \rangle, \quad (10)$$

where, $\mathcal{T}$ is the time-ordered product of two currents, and $J(x)$ is the interpolating currents for these states.

We model the tetraquarks X_{4c} and X_{4b} as structures formed by the axial-vector diquark $Q^T C \gamma_\mu Q$ and axial-vector antidiquark $\bar{Q} \gamma_\mu C \bar{Q}^T$. Corresponding interpolating current is given by the formula

$$J(x) = Q_a^T(x) C \gamma_\mu Q_b(x) \bar{Q}_a(x) \gamma^\mu C \bar{Q}_b^T(x), \quad (11)$$

where a , and b are color indices. In Eq. (11) $Q(x)$ denotes either c or b quark fields, and C is the charge conjugation matrix. The current $J(x)$ describes the tetraquark with spin-parities $J^{PC} = 0^{++}$.

In what follows, we write down formulas for the tetraquark X_{4c} . Expressions for the state X_{4b} can be obtained from them trivially. The physical side of the sum rule $\Pi^{\text{Phys}}(p)$

$$\Pi^{\text{Phys}}(p) = \frac{\langle 0 | J | X_{4c}(p) \rangle \langle X_{4c}(p) | J^\dagger | 0 \rangle}{m^2 - p^2} + \dots, \quad (12)$$

is derived from Eq. (10) by inserting a complete set of intermediate states with quark content and spin-parities of the tetraquark X_{4c} , and performing integration over x . Let us note that in $\Pi^{\text{Phys}}(p)$ the ground-state term is written down explicitly, whereas contributions of higher resonances and continuum states are shown by the dots.

The correlation function $\Pi^{\text{Phys}}(p)$ can be simplified using the matrix element

$$\langle 0 | J | X_{4c}(p) \rangle = f m, \quad (13)$$

which leads to the following expression

$$\Pi^{\text{Phys}}(p) = \frac{f^2 m^2}{m^2 - p^2} + \dots. \quad (14)$$

The correlator $\Pi^{\text{Phys}}(p)$ has simple Lorentz structure proportional to I , therefore the invariant amplitude $\Pi^{\text{Phys}}(p^2)$ is given by right-hand side of Eq. (14).

The QCD side of the sum rule $\Pi^{\text{OPE}}(p)$ has to be computed in the operator product expansion (OPE) with certain accuracy. For these purposes, one substitutes the current $J(x)$ into the correlator $\Pi(p)$, contracts relevant quark fields, and replaces contractions by the heavy quark propagators. These manipulations lead to the formula

$$\begin{aligned} \Pi^{\text{OPE}}(p) = i \int d^4 x e^{ipx} & \left\{ \text{Tr} \left[\gamma_\mu \tilde{S}_c^{b'b}(-x) \gamma_\nu S_c^{a'a}(-x) \right] \left[\text{Tr} \left[\gamma^\nu \tilde{S}_c^{aa'}(x) \gamma^\mu S_c^{bb'}(x) \right] \right. \right. \\ & - \text{Tr} \left[\gamma^\nu \tilde{S}_c^{ba'}(x) \gamma^\mu S_c^{ab'}(x) \right] \left. \right] + \text{Tr} \left[\gamma_\mu \tilde{S}_c^{a'b}(-x) \gamma_\nu S_c^{b'a}(-x) \right] \\ & \times \left[\text{Tr} \left[\gamma^\nu \tilde{S}_c^{ba'}(x) \gamma^\mu S_c^{ab'}(x) \right] - \text{Tr} \left[\gamma^\nu \tilde{S}_c^{aa'}(x) \gamma^\mu S_c^{bb'}(x) \right] \right] \Big\}, \end{aligned} \quad (15)$$

where

$$\tilde{S}_c(x) = C S_c^T(x) C, \quad (16)$$

with $S_c(x)$ being the c -quark propagator. The explicit expression of the heavy quark propagator $S_Q(x)$ can be found in Appendix.

In the case under analysis, the QCD side of the sum rules depends exclusively on the propagators of heavy quarks. The heavy quark propagator $S_Q^{ab}(x)$ apart from a perturbative term contains also components which are linear and quadratic in the gluon field strength. It does not depend on light quark or mixed quark-gluon vacuum condensates which are sources of main nonperturbative contributions to correlation functions.

The $\Pi^{\text{OPE}}(p)$ has simple Lorentz structure $\sim I$ as well. In what follows, the corresponding invariant amplitude will be denoted by $\Pi^{\text{OPE}}(p^2)$. Having equated two functions $\Pi^{\text{Phys}}(p^2)$ and $\Pi^{\text{OPE}}(p^2)$, applied the Borel transformation to suppress contributions of higher resonances and continuum states, and subtracted these contributions by employing the assumption about quark-hadron duality [55,56], we find the required sum rules for the mass and coupling of the tetraquark X_{4c} .

Calculation of the function $\Pi^{\text{OPE}}(p^2)$ is a next step in our efforts to derive the sum rules for m and f . Analyses demonstrate that after the Borel transformation and continuum subtraction the amplitude $\Pi(M^2, s_0)$ has the form

$$\Pi(M^2, s_0) = \int_{16m_c^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}. \quad (17)$$

Here, $\rho^{\text{OPE}}(s)$ is a two-point spectral density, which is found as an imaginary part of the invariant amplitude $\Pi^{\text{OPE}}(p^2)$. The function $\rho^{\text{OPE}}(s)$ contains a perturbative term $\rho^{\text{pert.}}(s)$ and a dimension-4 nonperturbative contribution proportional to $\langle \alpha_s G^2/\pi \rangle$. In Appendix, we write down the analytical expression for $\rho^{\text{pert.}}(s)$, and refrain from presenting a dimension-4 term which is rather lengthy.

Then, the sum rules for m and f are given by the formulas

$$m^2 = \frac{\Pi'(M^2, s_0)}{\Pi(M^2, s_0)} \quad (18)$$

and

$$f^2 = \frac{e^{m^2/M^2}}{m^2} \Pi(M^2, s_0), \quad (19)$$

respectively. In Eq. (18), we use the notation $\Pi'(M^2, s_0) = d\Pi(M^2, s_0)/d(-1/M^2)$.

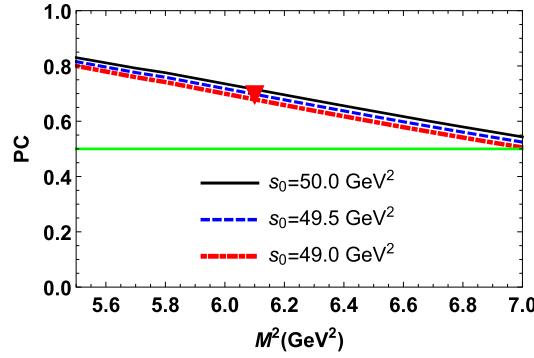


Fig. 1. The pole contribution PC as a function of the Borel parameter M^2 at different s_0 . The limit $PC = 0.5$ is shown by the horizontal line. The red triangle shows the point, where the mass m of the tetraquark X_{4c} has been extracted from the sum rule.

The sum rules Eqs. (18) and (19) depend on the gluon vacuum condensate and on masses of c and b quarks, numerical values of which are listed below

$$\begin{aligned} \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle &= (0.012 \pm 0.004) \text{ GeV}^4, \\ m_c &= (1.27 \pm 0.02) \text{ GeV}, \\ m_b &= 4.18^{+0.03}_{-0.02} \text{ GeV}. \end{aligned} \quad (20)$$

Choosing working windows for parameters M^2 and s_0 is another problem of the sum rule computations. They should be fixed in such a way that to meet a constraint imposed on the pole contribution (PC), and ensure convergence of the operator product expansion. Because, in the present article we consider only a nonperturbative term $\sim \langle \alpha_s G^2 / \pi \rangle$, the pole contribution plays a decisive role in determining of M^2 and s_0 . To estimate PC, we use the expression

$$PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \quad (21)$$

and require fulfillment of the constraint $PC \geq 0.5$.

The PC is employed to fix the higher limit of the Borel parameter M^2 . The lower limit for M^2 is found from a stability of the sum rules' results under variation of M^2 , and from prevalence of the perturbative term. Two values of M^2 extracted by this method fix boundaries of the region where M^2 can be varied. Calculations for the tetraquark X_{4c} show that the intervals

$$M^2 \in [5.5, 7] \text{ GeV}^2, \quad s_0 \in [49, 50] \text{ GeV}^2, \quad (22)$$

are appropriate for the parameters M^2 and s_0 , and comply with limits on PC and nonperturbative term. Thus, at $M^2 = 7 \text{ GeV}^2$ the pole contribution is 0.51, whereas at $M^2 = 5.5 \text{ GeV}^2$ it becomes equal to 0.82. At the minimum of $M^2 = 5.5 \text{ GeV}^2$, contribution of the nonperturbative term is negative and forms 2% of the correlation function. To demonstrate dynamics of the pole contribution, Fig. 1 we plot PC as a function of M^2 at different s_0 . It is seen, that the pole contribution exceeds 0.5 for all values of the parameters M^2 and s_0 from Eq. (22).

We extract the mass m and coupling f of the tetraquark X_{4c} by calculating them at different M^2 and s_0 , and determining their mean values averaged over the regions Eq. (22). Our predictions for m and f read

$$\begin{aligned} m &= (6570 \pm 55) \text{ MeV}, \\ f &= (5.61 \pm 0.39) \times 10^{-2} \text{ GeV}^4. \end{aligned} \quad (23)$$

The results in Eq. (23) correspond to sum rules predictions at approximately middle point of the regions in Eq. (22), i.e., to predictions at the point $M^2 = 6.1 \text{ GeV}^2$ and $s_0 = 49.5 \text{ GeV}^2$, where the pole contribution is $PC \approx 0.70$. This fact guarantees the dominance of PC in the obtained results, and confirms ground-state nature of the tetraquark X_{4c} . Dependence of m on the parameters M^2 and s_0 is depicted in Fig. 2.

The mass m of the tetraquark X_{4c} obtained in present article nicely agrees with the mass of the resonance $X(6600)$ fixed by the ATLAS and CMS collaborations, and belong to the wide threshold enhancement $6.2 - 6.8 \text{ GeV}$ in $J/\psi J/\psi$ mass distribution seen by LHCb. Therefore, at this level of our knowledge, we consider the tetraquark X_{4c} as a candidate to the $X(6600)$ state. But, for more detailed comparisons with ATLAS and CMS data, and credible statements about its nature, we need to evaluate the full width of X_{4c} .

In the case of the tetraquark X_{4b} a similar analysis yields the following working intervals for the Borel and continuum subtraction parameters

$$\begin{aligned} M^2 &\in [17.5, 18.5] \text{ GeV}^2, \\ s_0 &\in [375, 380] \text{ GeV}^2. \end{aligned} \quad (24)$$

The pole contribution in the interval for M^2 changes within limits

$$0.72 \geq PC \geq 0.66. \quad (25)$$

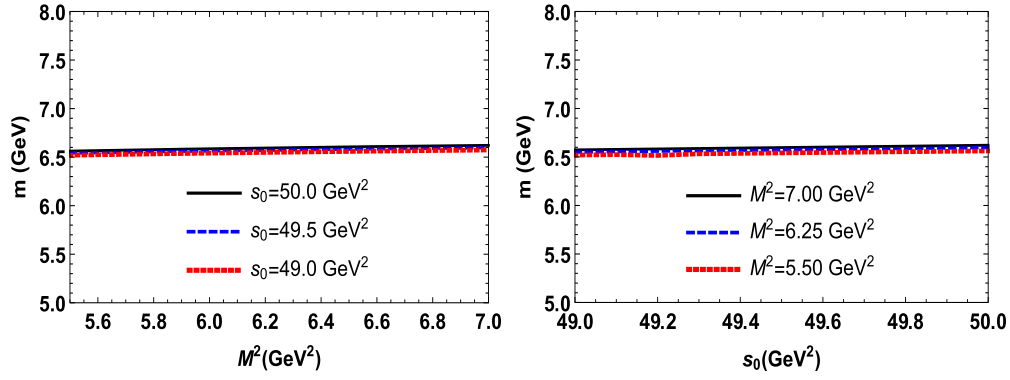


Fig. 2. Mass of the tetraquark X_{4c} as a function of the Borel parameter M^2 (left), and as a function of the continuum threshold s_0 (right).

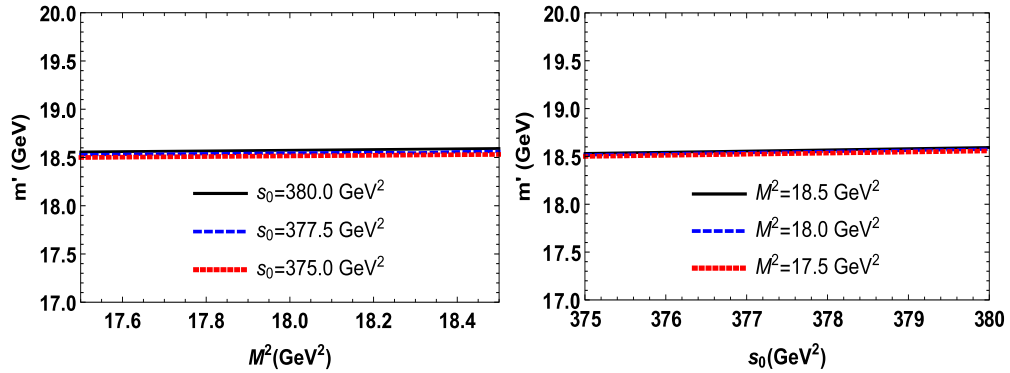


Fig. 3. The same as in Fig. 2, but for the mass m' of the tetraquark X_{4b} .

At $M^2 = 17.5 \text{ GeV}^2$ the dimension-4 term constitutes $\simeq -1.5\%$ of the result. The mass and current coupling of X_{4b} are

$$\begin{aligned} m' &= (18540 \pm 50) \text{ MeV}, \\ f' &= (6.1 \pm 0.4) \times 10^{-1} \text{ GeV}^4. \end{aligned} \quad (26)$$

Behavior of m' as a function of M^2 and s_0 is shown in Fig. 3.

Fully beauty scalar tetraquarks were investigated in numerous articles. The mass m' of the scalar $4b$ state was found equal to 18754 MeV, $(18826 \pm 25) \text{ MeV}$, and $18450 - 18590 \text{ MeV}$ in Refs. [7,8,10], respectively. These results were obtained by solving non-relativistic Schrodinger equation, using a phenomenological approach or the QCD sum rule method. An estimate 18750 MeV for the mass of the $4b$ ground-state particle was made in the context of a relativized diquark model with one-gluon-exchange and confining potentials [33]. A diffusion Monte Carlo method used to solve nonrelativistic many-body system led to the result $m' = (18690 \pm 30) \text{ MeV}$ [34]. Considerably larger mass 19315 MeV was predicted for the $J^{PC} = 0^{++}$ diquark-antidiquark state in Ref. [50].

These results differ from each other not only quantitatively, but imply also different mechanisms for decays of these particles. Thus, there are two important thresholds for fully beauty tetraquarks, i.e., the $2\eta_b$ and $2\Upsilon(1S)$ thresholds that amount to 18798 MeV and 18921 MeV, respectively. Possible decay modes of $4b$ four-quark compounds to ordinary mesons and leptons are determined by their positions in this mass scale.

Our result $m' = 18540 \text{ MeV}$ for the mass of X_{4b} is consistent with prediction of Ref. [10] calculated also in the framework of the sum rule method. It is below the lowest $2\eta_b$ threshold in the sector of fully beauty ordinary mesons. In other words, X_{4b} is stable against strong decays to conventional $b\bar{b}$ mesons. Similar conclusions were drawn also in Refs. [7,10]. Such structures transform to conventional particles due to $b\bar{b}$ annihilation to a gluon or a light quark-antiquark pair, through two and three gluons produced by a $b\bar{b}$ pair which later are converted into light hadrons [44]. In Ref. [44] the width of the fully beauty tetraquark with the mass below the $2\eta_b$ threshold was estimated around of 8.5 MeV. Hence, the tetraquark X_{4b} has a finite width though it does not fall apart to $2\eta_b$ and $2\Upsilon(1S)$ final states, but processes that generate this width are beyond the scope of the present work.

3. Decay $X_{4c} \rightarrow J/\psi J/\psi$

The mass of the tetraquark X_{4c} exceeds the two-meson thresholds both in $J/\psi J/\psi$ and $\eta_c \eta_c$ channels, therefore S -wave processes $X_{4c} \rightarrow J/\psi J/\psi$ and $X_{4c} \rightarrow \eta_c \eta_c$ are allowed decay modes of this particle. Another channel which will be considered in the present article is P -wave decay mode $X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$.

We begin our investigations from analysis of the process $X_{4c} \rightarrow J/\psi J/\psi$. The partial width of this decay is determined by the strong coupling g_1 of the particles at the vertex $X_{4c} J/\psi J/\psi$. In the context of the QCD sum rule method g_1 can be extracted from the three-point correlation function

$$\Pi_{\mu\nu}(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | T \{ J_\mu^{J/\psi}(y) J_\nu^{J/\psi}(0) J^\dagger(x) \} | 0 \rangle, \quad (27)$$

where $J_\mu^{J/\psi}(x)$ is the interpolating currents for the J/ψ meson. The $J(x)$ is given by Eq. (11), while for $J_\mu^{J/\psi}(x)$ we use

$$J_\mu^{J/\psi}(x) = \bar{c}_i(x) \gamma_\mu c_i(x), \quad (28)$$

where $i = 1, 2, 3$ are the color indices. The 4-momentum of the tetraquark X_{4c} is p , whereas momenta of the J/ψ mesons are p' and $q = p - p'$, respectively.

We follow the standard prescriptions of the sum rule method and express the correlation function $\Pi_{\mu\nu}(p, p')$ in terms of involved particles' phenomenological parameters. Isolating the ground-state contribution to the correlation function (27) from effects of higher resonances and continuum states, for the physical side of the sum rule $\Pi_{\mu\nu}^{\text{Phys}}(p, p')$, we get

$$\Pi_{\mu\nu}^{\text{Phys}}(p, p') = \frac{\langle 0 | J_\mu^{J/\psi} | J/\psi(p') \rangle \langle 0 | J_\nu^{J/\psi} | J/\psi(q) \rangle \langle J/\psi(p') J/\psi(q) | X_{4c}(p) \rangle \langle X_{4c}(p) | J^\dagger | 0 \rangle}{p'^2 - m_1^2} \frac{q^2 - m_1^2}{q^2 - m_1^2} + \dots, \quad (29)$$

with m_1 being the mass of the J/ψ meson.

The function $\Pi_{\mu\nu}^{\text{Phys}}(p, p')$ can be simplified by employing the matrix elements of the tetraquark X_{4c} and J/ψ meson. The matrix element of X_{4c} is given by Eq. (13), whereas for $\langle 0 | J_\mu^{J/\psi} | J/\psi(p) \rangle$ we use

$$\langle 0 | J_\mu^{J/\psi} | J/\psi(p) \rangle = f_1 m_1 \varepsilon_\mu(p), \quad (30)$$

where f_1 and ε_μ are the decay constant and polarization vector of the J/ψ meson, respectively. We also model the vertex $\langle J/\psi(p') J/\psi(q) | X_{4c}(p) \rangle$ by the expression

$$\langle J/\psi(p') J/\psi(q) | X_{4c}(p) \rangle = g_1(q^2) [q \cdot p' \varepsilon^*(p') \cdot \varepsilon^*(q) - q \cdot \varepsilon^*(p') p' \cdot \varepsilon^*(q)], \quad (31)$$

which has the gauge-invariant form.

After these transformations $\Pi_{\mu\nu}^{\text{Phys}}(p, p')$ is given by the formula

$$\Pi_{\mu\nu}^{\text{Phys}}(p, p') = g_1(q^2) \frac{f m f_1^2 m_1^2}{(p^2 - m^2)(p'^2 - m_1^2)(q^2 - m_1^2)} \left[\frac{1}{2} (m^2 - m_1^2 - q^2) g_{\mu\nu} - q_\mu p'_\nu \right] + \dots, \quad (32)$$

where the ellipses stand for contributions of higher resonances and continuum states. The correlator Eq. (32) contains different Lorentz structures, which may be used to construct the sum rule for $g_1(q^2)$. We choose to work with the term $\sim g_{\mu\nu}$ and denote the relevant invariant amplitude by $\Pi^{\text{Phys}}(p^2, p'^2, q^2)$.

The correlation function $\Pi_{\mu\nu}(p, p')$ calculated in terms of heavy quark propagators reads

$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(p, p') &= -2i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \left\{ \text{Tr} \left[\gamma_\mu S_c^{ib}(y-x) \gamma_\alpha \tilde{S}_c^{ja}(-x) \gamma_\nu \tilde{S}_c^{bj}(x) \gamma^\alpha S_c^{ai}(x-y) \right] \right. \\ &\quad \left. - \text{Tr} \left[\gamma_\mu S_c^{ia}(y-x) \gamma_\alpha \tilde{S}_c^{jb}(-x) \gamma_\nu \tilde{S}_c^{bj}(x) \gamma^\alpha S_c^{ai}(x-y) \right] \right\}. \end{aligned} \quad (33)$$

The invariant amplitude $\Pi^{\text{OPE}}(p^2, p'^2, q^2)$ which corresponds to the term $\sim g_{\mu\nu}$ in Eq. (33) constitutes the QCD side of the sum rule. Having equated these two invariant amplitudes, carried out the doubly Borel transformations over variables p^2 and p'^2 and performed continuum subtraction, one finds the sum rule for $g_1(q^2)$

$$g_1(q^2) = \frac{2}{f m f_1^2 m_1^2} \frac{q^2 - m_1^2}{m^2 - m_1^2 - q^2} e^{m^2/M_1^2} e^{m_1^2/M_2^2} \Pi(\mathbf{M}^2, \mathbf{s}_0, q^2). \quad (34)$$

Here, $\Pi(\mathbf{M}^2, \mathbf{s}_0, q^2)$ is the amplitude $\Pi^{\text{OPE}}(p^2, p'^2, q^2)$ after the Borel transformation and subtraction procedures: It can be expressed in terms of the spectral density $\rho(s, s', q^2)$ calculated as an imaginary part of relevant component of the correlation function $\Pi_{\mu\nu}^{\text{OPE}}(p, p')$,

$$\Pi(\mathbf{M}^2, \mathbf{s}_0, q^2) = \int_{16m_c^2}^{s_0} ds \int_{4m_c^2}^{s'_0} ds' \rho(s, s', q^2) e^{-s/M_1^2} e^{-s'/M_2^2}, \quad (35)$$

where $\mathbf{M}^2 = (M_1^2, M_2^2)$ and $\mathbf{s}_0 = (s_0, s'_0)$ are the Borel and continuum threshold parameters, respectively.

The form factor $g_1(q^2)$ depends on the masses and current couplings (decay constant) of the tetraquark X_{4c} and the meson J/ψ which appear in the numerical computations as input parameters. Their values are moved to Table 1, which contains also spectroscopic parameters of η_c and $\chi_{c1}(1P)$ mesons required to investigate two other decays of X_{4c} . The masses all of the mesons are borrowed from Ref. [57]. For the decay constant of the meson J/ψ , we employ the experimental value reported in Ref. [58]. As f_{η_c} and $f_{\chi_{c1}}$, we use predictions made in Refs. [59,60] on the basis of the sum rule method, respectively.

To carry out numerical computations it is necessary also to choose the working regions for the parameters $\mathbf{M}^2$ and $\mathbf{s}_0$. The constraints imposed on $\mathbf{M}^2$ and $\mathbf{s}_0$ are standard restrictions of the sum rule calculations and were explained in the previous section. For M_1^2 and s_0 , associated with the X_{4c} channel, we use the working windows from Eq. (22). The parameters (M_2^2, s'_0) for the J/ψ channel are changed inside borders

$$M_2^2 \in [4, 5] \text{ GeV}^2, \quad s'_0 \in [12, 13] \text{ GeV}^2. \quad (36)$$

Table 1
Masses and decay constants of $\bar{c}c$ mesons, which have been used in numerical computations.

Parameters	Values (in MeV units)
$m_1[m_{J/\psi}]$	3096.900 ± 0.006
$f_1[f_{J/\psi}]$	409 ± 15
$m_2[m_{\eta_c}]$	2983.9 ± 0.4
$f_2[f_{\eta_c}]$	320 ± 40
$m_3[m_{\chi_{c1}}]$	3510.67 ± 0.05
$f_3[f_{\chi_{c1}}]$	344 ± 27

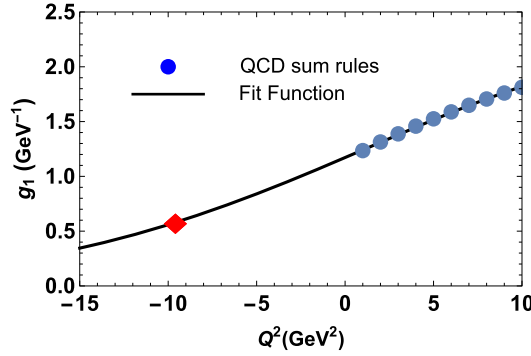


Fig. 4. The sum rule predictions and fit function for the strong coupling $g_1(Q^2)$. The red diamond denotes the point $Q^2 = -m_1^2$.

It is known that the sum rule method leads to reliable predictions in the deep-Euclidean region $q^2 < 0$. For our purposes, it is convenient to introduce a new variable $Q^2 = -q^2$ and denote the obtained function by $g_1(Q^2)$. A range of Q^2 studied by the sum rule analysis covers the region $Q^2 = 1 - 10 \text{ GeV}^2$. The results of calculations are plotted in Fig. 4. But the width of the decay $X_{4c} \rightarrow J/\psi J/\psi$ is determined by the form factor $g_1(q^2)$ at the mass shell $q^2 = m_1^2$. Stated differently, one has to find $g_1(Q^2 = -m_1^2)$.

To solve this problem, we use a fit function $\mathcal{G}_1(Q^2)$ that at momenta $Q^2 > 0$ gives the same values as the sum rule calculations, but can be extrapolated to the region of $Q^2 < 0$. In this paper, we employ the functions $\mathcal{G}_i(Q^2)$, $i = 1, 2, 3$

$$\mathcal{G}_i(Q^2) = \mathcal{G}_i^0 \exp \left[c_i^1 \frac{Q^2}{m^2} + c_i^2 \left(\frac{Q^2}{m^2} \right)^2 \right], \quad (37)$$

with parameters $\mathcal{G}_i^0$, c_i^1 and c_i^2 .

Calculations prove that $\mathcal{G}_1^0 = 1.17 \text{ GeV}^{-1}$, $c_1^1 = 2.55$, and $c_1^2 = -2.79$ give nice agreement with the sum rule's data for $g_1(Q^2)$ shown in Fig. 4. At the mass shell $q^2 = m_1^2$ the function $\mathcal{G}_1(Q^2)$ is equal to

$$g_1 \equiv \mathcal{G}_1(-m_1^2) = (5.8 \pm 1.2) \times 10^{-1} \text{ GeV}^{-1}. \quad (38)$$

The partial width of the process $X_{4c} \rightarrow J/\psi J/\psi$ can be obtained by employing the following expression

$$\Gamma[X_{4c} \rightarrow J/\psi J/\psi] = g_1^2 \frac{\lambda}{8\pi} \left(\frac{m_1^4}{m^2} + \frac{2\lambda^2}{3} \right), \quad (39)$$

where $\lambda = \lambda(m, m_1, m_1)$ and

$$\lambda(a, b, c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)}}{2a}. \quad (40)$$

Then it is easy to find

$$\Gamma[X_{4c} \rightarrow J/\psi J/\psi] = (43 \pm 13) \text{ MeV}. \quad (41)$$

4. Processes $X_{4c} \rightarrow \eta_c \eta_c$ and $X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$

The decays $X_{4c} \rightarrow \eta_c \eta_c$ and $X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$ can be explored in a similar manner. The strong coupling g_2 that describes the vertex $X_{4c} \eta_c \eta_c$ can be extracted from the correlation function

$$\Pi(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J^{\eta_c}(y) J^{\eta_c}(0) J^\dagger(x) \} | 0 \rangle, \quad (42)$$

where the current $J^{\eta_c}(x)$ is

$$J^{\eta_c}(x) = \bar{c}_i(x) i \gamma_5 c_i(x). \quad (43)$$

Separating from each other the ground-state contribution and effects of higher resonances and continuum states, we write the correlation function (42) in the following form

$$\Pi^{\text{Phys}}(p, p') = \frac{\langle 0 | J^{\eta_c} | \eta_c(p') \rangle \langle 0 | J^{\eta_c} | \eta_c(q) \rangle}{p'^2 - m_2^2} \frac{\langle \eta_c(p') \eta_c(q) | X_{4c}(p) \rangle}{q^2 - m_2^2} \frac{\langle X_{4c}(p) | J^\dagger | 0 \rangle}{p^2 - m^2} + \dots, \quad (44)$$

where m_2 is the mass of the η_c meson. We define the vertex composed of a scalar and two pseudoscalar particles by means of the formula

$$\langle \eta_c(p') \eta_c(q) | X_{4c}(p) \rangle = g_2(q^2) p \cdot p'. \quad (45)$$

To express the correlator $\Pi^{\text{Phys}}(p, p')$ in terms of physical parameters of particles X_{4c} and η_c , we use the matrix element Eq. (13) and

$$\langle 0 | J^{\eta_c} | \eta_c \rangle = \frac{f_2 m_2^2}{2m_c}, \quad (46)$$

with f_2 being the decay constant of the η_c meson. Then, the correlation function $\Pi^{\text{Phys}}(p, p')$ takes the form

$$\Pi^{\text{Phys}}(p, p') = g_2(q^2) \frac{f m f_2^2 m_2^4}{4m_c^2 (p^2 - m^2) (p'^2 - m_2^2)} \frac{m^2 + m_2^2 - q^2}{2(q^2 - m_2^2)} + \dots. \quad (47)$$

The function $\Pi^{\text{Phys}}(p, p')$ has a Lorentz structure that is proportional to I, hence rhs of Eq. (47) is the corresponding invariant amplitude $\tilde{\Pi}^{\text{Phys}}(p^2, p'^2, q^2)$.

Using the heavy quark propagators, we can find the QCD side of the sum rule

$$\begin{aligned} \Pi^{\text{OPE}}(p, p') &= 2i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \left\{ \text{Tr} \left[\gamma_5 S_c^{ia}(y-x) \gamma_\alpha \tilde{S}_c^{jb}(-x) \gamma_5 \tilde{S}_c^{bj}(x) \gamma^\alpha S_c^{ai}(x-y) \right] \right. \\ &\quad \left. - \text{Tr} \left[\gamma_5 S_c^{ia}(y-x) \gamma_\alpha \tilde{S}_c^{jb}(-x) \gamma_5 \tilde{S}_c^{aj}(x) \gamma^\alpha S_c^{bi}(x-y) \right] \right\}. \end{aligned} \quad (48)$$

The sum rule for the strong form factor $g_2(q^2)$ equals to

$$g_2(q^2) = \frac{8m_c^2}{f m f_2^2 m_2^4} \frac{q^2 - m_2^2}{m^2 + m_2^2 - q^2} e^{m^2/M_1^2} e^{m_2^2/M_2^2} \tilde{\Pi}(\mathbf{M}^2, \mathbf{s}_0, q^2), \quad (49)$$

with $\tilde{\Pi}(\mathbf{M}^2, \mathbf{s}_0, q^2)$ being the invariant amplitude $\tilde{\Pi}^{\text{OPE}}(p^2, p'^2, q^2)$ corresponding to the correlator $\Pi^{\text{OPE}}(p, p')$ after the Borel transformations and continuum subtractions.

We carry out numerical computations using Eq. (49), parameters of the meson η_c from Table 1, and working regions for $\mathbf{M}^2$ and $\mathbf{s}_0$. The Borel and continuum subtraction parameters M_1^2 and s_0 in the X_{4c} channel is chosen as in Eq. (22), whereas for M_2^2 and s'_0 which correspond to the η_c channel, we employ

$$M_2^2 \in [3.5, 4.5] \text{ GeV}^2, \quad s'_0 \in [11, 12] \text{ GeV}^2. \quad (50)$$

The interpolating function $\mathcal{G}_2(Q^2)$ has the parameters $\mathcal{G}_2^0 = 0.65 \text{ GeV}^{-1}$, $c_2^1 = 3.19$, and $c_2^2 = -3.34$. For the strong coupling g_2 , we get

$$g_2 \equiv \mathcal{G}_2(-m_2^2) = (2.9 \pm 0.6) \times 10^{-1} \text{ GeV}^{-1}. \quad (51)$$

The width of the process $X_{4c} \rightarrow \eta_c \eta_c$ is determined by means of the formula

$$\Gamma[X_{4c} \rightarrow \eta_c \eta_c] = g_2^2 \frac{m_2^2 \tilde{\lambda}}{8\pi} \left(1 + \frac{\tilde{\lambda}^2}{m_2^2} \right), \quad (52)$$

where $\tilde{\lambda} = \lambda(m, m_2, m_2)$. Finally, we obtain

$$\Gamma[X_{4c} \rightarrow \eta_c \eta_c] = (51 \pm 15) \text{ MeV}. \quad (53)$$

Treatment of the P -wave decay $X_{4c} \rightarrow \eta_c \chi_{c1}(P)$ does not generate additional technical details, and is performed in a usual manner. The three-point correlator to be considered in this case is

$$\Pi_\mu(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J_\mu^{\chi_{c1}}(y) J^{\eta_c}(0) J^\dagger(x) | 0 \rangle \}, \quad (54)$$

where $J_\mu^{\chi_{c1}}(y)$ is the interpolating current for the meson $\chi_{c1}(1P)$

$$J_\mu^{\chi_{c1}}(y) = \bar{c}_j(x) \gamma_5 \gamma_\mu c_j(x). \quad (55)$$

In terms of the physical parameters of the particles the correlation function has the form

$$\Pi_\mu^{\text{Phys}}(p, p') = g_3(q^2) \frac{f m f_2 m_2^2 f_3 m_3}{2m_c (p^2 - m^2) (p'^2 - m_3^2)} \frac{1}{q^2 - m_2^2} \left[\frac{m^2 - m_3^2 - q^2}{2m_3^2} p'_\mu - q_\mu \right] + \dots. \quad (56)$$

Table 2

Decay channels of the tetraquark X_{4c} , strong couplings g_i , and partial widths Γ_i . The coupling g_3 marked by a star is dimensionless.

i	Channels	g_i (GeV^{-1})	Γ_i (MeV)
1	$X_{4c} \rightarrow J/\psi J/\psi$	$(5.8 \pm 1.2) \times 10^{-1}$	43 ± 13
2	$X_{4c} \rightarrow \eta_c \eta_c$	$(2.9 \pm 0.6) \times 10^{-1}$	51 ± 15
3	$X_{4c} \rightarrow \eta_c \chi_{c1}(1P)$	$10.9 \pm 2.8^*$	16 ± 6

In Eq. (56) m_3 and f_3 are the mass and decay constant of the meson $\chi_{c1}(1P)$. To derive the correlator $\Pi_\mu^{\text{Phys}}(p, p')$, we have used the known matrix elements of the tetraquark X_{4c} and meson η_c , as well as new matrix elements

$$\langle 0 | J_\mu^{\chi_{c1}} | \chi_{c1}(p') \rangle = f_3 m_3 \varepsilon_\mu^*(p'), \quad (57)$$

and

$$\langle \eta_c(q) \chi_{c1}(p') | X_{4c}(p) \rangle = g_3(q^2) p \cdot \varepsilon^*(p'), \quad (58)$$

where $\varepsilon_\mu^*(p')$ is the polarization vector of $\chi_{c1}(1P)$.

The QCD side $\Pi_\mu^{\text{OPE}}(p, p')$ is given by the formula

$$\begin{aligned} \Pi_\mu^{\text{OPE}}(p, p') = 2i^3 \int d^4x d^4y e^{ip'y} e^{-ipx} \left\{ \text{Tr} \left[\gamma_\mu \gamma_5 S_c^{ia}(y-x) \gamma_\alpha \tilde{S}_c^{jb}(-x) \gamma_5 \tilde{S}_c^{bj}(x) \gamma^\alpha S_c^{ai}(x-y) \right] \right. \\ \left. - \text{Tr} \left[\gamma_\mu \gamma_5 S_c^{ia}(y-x) \gamma_\alpha \tilde{S}_c^{jb}(-x) \gamma_5 \tilde{S}_c^{aj}(x) \gamma^\alpha S_c^{bi}(x-y) \right] \right\}. \end{aligned} \quad (59)$$

The sum rule for $g_3(q^2)$ is derived using the invariant amplitudes corresponding to terms $\sim p'_\mu$ in both $\Pi_\mu^{\text{Phys}}(p, p')$ and $\Pi_\mu^{\text{OPE}}(p, p')$. In numerical analysis, M_2^2 and s'_0 in the χ_{c1} channel are chosen in the following way

$$M_2^2 \in [4, 5] \text{ GeV}^2, \quad s'_0 \in [13, 14] \text{ GeV}^2. \quad (60)$$

For the parameters of the fit function $\mathcal{G}_3(Q^2)$, we get $\mathcal{G}_3^0 = 24.08$, $c_3^1 = 2.98$, and $c_3^2 = -4.26$. Then, the strong coupling g_3 is equal to

$$g_3 \equiv \mathcal{G}_3(-m_2^2) = 10.9 \pm 2.8. \quad (61)$$

The width of the decay $X_{4c} \rightarrow \eta_c \chi_{c1}(P)$ can be calculated by means of the expression

$$\Gamma[X_{4c} \rightarrow \eta_c \chi_{c1}(P)] = g_3^2 \frac{\hat{\lambda}^3}{24\pi m_3^2}, \quad (62)$$

where $\hat{\lambda} = \lambda(m, m_2, m_3)$. For the width of this process, we obtain the estimate:

$$\Gamma[X_{4c} \rightarrow \eta_c \chi_{c1}(P)] = (16 \pm 6) \text{ MeV}. \quad (63)$$

The widths all of three decays are collected in Table 2. Based on these results, it is not difficult to find that

$$\Gamma_{4c} = (110 \pm 21) \text{ MeV}, \quad (64)$$

which nicely agrees with CMS datum Γ_1^{CMS} .

5. Discussion and concluding notes

In the present article, we have performed detailed analysis of the tetraquark X_{4c} by calculating the mass m and full width Γ_{4c} of this scalar diquark-antidiquark state. Our findings are in agreements with the experimental data $m_1^{\text{CMS}} = (6552 \pm 10 \pm 12) \text{ MeV}$ and $\Gamma_1^{\text{CMS}} = (124^{+32}_{-26} \pm 33) \text{ MeV}$ of the CMS Collaboration. The mass of X_{4c} is compatible also with m_1^{ATL} if one takes into account existing experimental and theoretical errors. We have interpreted the ground-level 1S tetraquark X_{4c} built of axial-vector constituents as the resonance $X(6600)$.

The partial width of the decay $X_{4c} \rightarrow \eta_c \eta_c$ is comparable with $\Gamma[X_{4c} \rightarrow J/\psi J/\psi]$. The new fully charmed resonances were observed in the di- J/ψ mass distribution through 4μ final states. It is known that decays to lepton pairs e^+e^- and $\mu^+\mu^-$ are among important modes of the J/ψ meson [57]. But, the η_c meson's main channels are decays to hadronic resonances, for example, to $\rho\rho$ mesons. Naturally, the process $X_{4c} \rightarrow \eta_c \eta_c$ could not be seen in 4μ events.

There are numerous publications, in which properties of the tetraquark X_{4c} were studied using various methods (for complete list of relevant publications see, Ref. [50]). These investigations intensified after discovery of resonances $X(6200)$, $X(6600)$, $X(6900)$ and $X(7300)$. Comparing our result for the mass of X_{4c} with $(6.46 \pm 0.16) \text{ GeV}$ and $6.46^{+0.13}_{-0.17} \text{ GeV}$ from Refs. [10,39], we see that though m exceeds them, within ambiguities of calculations all predictions are comparable with each other. But what is more important, decays to $J/\psi J/\psi$ pairs are kinematically allowed channels for these structures.

The first resonance $X(6200)$ in the list of the fully charmed states may be a manifestation of the hadronic molecule $\eta_c \eta_c$ in the $J/\psi J/\psi$ spectrum. But to be detected the mass of $\eta_c \eta_c$ must exceed the di- J/ψ threshold $\simeq 6195 \text{ MeV}$. In Ref. [42] the authors predicted $M_{\eta_c \eta_c} = 6029 \pm 198 \text{ MeV}$ that in upper limit overshoots the di- J/ψ threshold. Alternatively, appearance of the near-threshold state $X(6200)$ may be explained by coupled-channel effects [46].

The next structure, $X(6900)$, can be considered in the diquark-antidiquark model provided it composed of pseudoscalar components. In fact, the mass of such tetraquark was estimated around (6.82 ± 0.18) GeV and (6.80 ± 0.27) GeV in Refs. [10,42], respectively. The hadronic molecule $\chi_{c0}\chi_{c0}$ with the mass $\simeq 6.93$ GeV is an alternative candidate to the resonance $X(6900)$ [42].

More detailed analyses of assumptions about a diquark-antidiquark or hadronic molecule nature of the resonances $X(6200)$ and $X(6900)$ were performed in our articles [61,62]. In these works, we applied the sum rule method to investigate the diquark-antidiquark state T_{4c} built of pseudoscalar constituents $c_a^T C c_b$ and $\bar{c}_a C \bar{c}_b^T$, as well as hadronic molecules $\eta_c \eta_c$ and $\chi_{c0} \chi_{c0}$. In Ref. [62] it was demonstrated that the molecule $\eta_c \eta_c$ with the mass (6264 ± 50) MeV and full width (320 ± 72) MeV is a natural candidate to the resonance $X(6200)$. Our prediction for the mass of this molecule is larger than $M_{\eta_c \eta_c}$, but has some overlapping region with it.

The mass of the tetraquark T_{4c} amounts to (6928 ± 50) MeV and is compatible with previous sum rule predictions and relevant LHCb-ATLAS-CMS data, especially with the result of the CMS Collaboration for $X(6900)$ [61]. The full width of T_{4c} was evaluated by taking into account its allowed decay channels and found equal to (128 ± 22) MeV in agreement with the CMS measurements. The parameters of the molecule $\chi_{c0} \chi_{c0}$ are equal to (6954 ± 50) MeV and (138 ± 18) MeV, respectively [62]. It may also be interpreted as a resonance $X(6900)$, or considered as its part in the tetraquark-molecule mixing model.

As is seen, though diquark-antidiquark states and hadronic molecules have different internal organizations, both of them may be used to model X resonances. Such “universality” of the X structures is connected mainly with errors of measurements reported by different collaborations. To make a choice between different models for X particles, one needs more precise data on their parameters.

The heaviest state $X(7300)$ from this list is presumably a radially excited $X_{4c}(2S)$ tetraquark. An argument in favor of such assumption came from the ATLAS Collaboration, which fixed the resonances $X(6600)$ and $X(7300)$ in the $J/\psi J/\psi$ and $J/\psi \psi'$ mass distributions, respectively. In other words

$$\begin{aligned} X(7300) &\rightarrow J/\psi \psi', \\ X(6600) &\rightarrow J/\psi J/\psi, \end{aligned} \quad (65)$$

are decay modes of these resonances. The mass gap between ψ' and J/ψ is around 590 MeV, whereas for $X(7300)$ and $X(6600)$ the mass difference equals to 600 MeV (ATLAS) and 735 MeV (CMS). Then, it is natural to suppose that $X(7300)$ is the first radially excited state of $X(6600)$. Originally, similar hypothesis was made in Ref. [63], while considering the main decay channels of the resonances $Z_c(3900)$ and $Z_c(4330)$:

$$\begin{aligned} Z_c(4330) &\rightarrow \psi' \pi, \\ Z_c(3900) &\rightarrow J/\psi \pi. \end{aligned} \quad (66)$$

It was supposed that $Z_c(4330)$ is first radial excitation of the tetraquark $Z_c(3900)$. This idea was later confirmed by calculations carried out using the diquark-antidiquark model and sum rule method [64,65]. In light of this analysis the assumption about $2S$ excited nature of $X(7300)$ looks plausible. Results of our investigations seem support this assumption and will be reported very soon.

We have calculated also the mass of the fully beauty scalar state X_{4b} . It turned out that, its mass $m' = (18540 \pm 50)$ MeV is smaller than the $\eta_b \eta_b$ threshold, and hence X_{4b} does not decay to a pair of hidden-bottom mesons and cannot be observed in $\eta_b \eta_b$ or $\Upsilon(1S) \Upsilon(1S)$ mass distributions. The stability of X_{4b} in these channels was already predicted in Refs. [7,10]. Its transformation to ordinary mesons can proceed through subprocesses $\bar{b}b \rightarrow \bar{q}q(\bar{s}s)$ and $\bar{b}b \rightarrow 2g(3g)$ that result in the decay $X_{4b} \rightarrow B^+ B^-$ and other similar processes [44]. The weak leptonic and nonleptonic decays of X_{4b} are also among its possible transitions to conventional mesons.

It is clear that controversial character of conclusions about nature of the fully heavy resonances is connected with different models and schemes employed for their investigations. In some of these articles, for instance, X_{4b} can decay to a pair of pseudoscalar mesons $\eta_b \eta_b$, but is stable against $\Upsilon(1S) \Upsilon(1S)$ mode, whereas in other publications X_{4b} is stable in both of these channels. In the case of fully charmed states a same resonance due to large experimental errors, may be interpreted within both the molecule and diquark-antidiquark models.

We would like to emphasize that a large part of conclusions about the ground-state and excited states X_{4c} and X_{4b} was drawn using information on masses of these structures. In our view, in scenarios with four-quark mesons one has to calculate also their widths, otherwise statements made by relying only on the masses of these structures remain not fully convincing.

Declaration of competing interest

We declare that there is no conflict of interests regarding this manuscript.

Data availability

No data was used for the research described in the article.

Appendix A. Heavy quark propagator $S_Q^{ab}(x)$ and spectral density $\rho^{\text{pert.}}(s, \alpha, \beta, \gamma)$

In the current article, for the heavy quark propagator $S_Q^{ab}(x)$ ($Q = c, b$), we employ

$$\begin{aligned} S_Q^{ab}(x) = i \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} &\left\{ \frac{\delta_{ab} (\not{k} + m_Q)}{k^2 - m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta}}{4} \frac{\sigma_{\alpha\beta} (\not{k} + m_Q) + (\not{k} + m_Q) \sigma_{\alpha\beta}}{(k^2 - m_Q^2)^2} \right. \\ &\left. + \frac{g_s^2 G^2}{12} \delta_{ab} m_Q \frac{k^2 + m_Q \not{k}}{(k^2 - m_Q^2)^4} + \dots \right\}. \end{aligned} \quad (A.1)$$

Here, we have used the notations

$$G_{ab}^{\alpha\beta} \equiv G_A^{\alpha\beta} \lambda_{ab}^A / 2, \quad G^2 = G_{\alpha\beta}^A G_A^{\alpha\beta}, \quad (A.2)$$

where $G_A^{\alpha\beta}$ is the gluon field-strength tensor, and λ^A are the Gell-Mann matrices. The indices A, B, C run in the range $1, 2, \dots, 8$.

The invariant amplitude $\Pi(M^2, s_0)$ obtained after the Borel transformation and subtraction procedures is given by the expression

$$\Pi(M^2, s_0) = \int_{16m_Q^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2},$$

where the spectral density $\rho^{\text{OPE}}(s)$ is determined by the formula

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert.}}(s) + \langle \alpha_s G^2 / \pi \rangle \rho^{\text{Dim4}}(s). \quad (A.3)$$

The components $\rho^{\text{pert.}}(s)$ and $\rho^{\text{Dim4}}(s)$ of the spectral density are

$$\rho^{\text{pert.}}(s) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma \rho^{\text{pert.}}(s, \alpha, \beta, \gamma), \quad (A.4)$$

where the variables α, β , and γ are Feynman parameters.

The function $\rho^{\text{pert.}}(s, \alpha, \beta, \gamma)$ has the form

$$\begin{aligned} \rho^{\text{pert.}}(s, \alpha, \beta, \gamma) = & \frac{\Theta(L_1) N_1^2}{64\pi^6 N_2^8 N_3^5 (1 - \gamma - \beta)^2} \left\{ -6m_Q^4 (\beta + \gamma - 1)^2 N_2^4 N_3^3 + m_Q^2 N_2^2 N_3 \left\{ 3Ls\alpha \left[N_2^2 (N_3 - L\alpha) \right. \right. \right. \\ & + LN_2\alpha\gamma(-N_3(2N_3 + (\beta + \gamma - 1)^2) + 4N_3\alpha(\beta + \gamma - 1) - 2\alpha^2(L^2 - 2N_3)) + L^2\alpha^2\gamma^2(N_3(N_3 + (\gamma + \beta - 1)^2) \\ & - 2N_3\alpha(\beta + \gamma - 1) + \alpha^2(L^2 - 2N_3)) \left. \right] + 2N_1 \left[-LN_3^2\alpha - L\alpha(\gamma(\beta + \gamma - 1) + \alpha(\gamma + \beta - 1) + \alpha^2) \right. \\ & \times (\beta(\gamma + \beta - 1) + \alpha(\beta + \gamma - 1) + \alpha^2) + N_3 \left(\gamma\beta(\beta + \gamma - 1)^2 + \alpha(\beta + \gamma - 1)^2 + \alpha^2(\beta + \gamma - 1)(2\beta + 2\gamma - 1) \right. \\ & + 4\alpha^3(\beta + \gamma - 1) + 2\alpha^4) \left. \right] \left. \right\} - 3L\alpha(L\alpha - N_3) \left\{ 2L^2s^2\alpha^2\gamma(N_3 - L\alpha)(N_2 - L\alpha\gamma) + 2LN_1s\alpha \left[2L^2\alpha^2\gamma^2(N_3 - L\alpha)^3 \right. \right. \\ & + N_2^2(N_3 + \gamma(\beta + \gamma - 1) - \alpha L) + LN_2\alpha\gamma(-3N_3 - \gamma(\beta + \gamma - 1) + 3\alpha L) + N_1^2(-LN_3\alpha + (\beta^2 + (\beta + \alpha)(\alpha + \gamma - 1)) \\ & \times (\gamma^2 + (\gamma + \alpha)(\alpha + \beta - 1))) \left. \right] \left. \right\}. \end{aligned} \quad (A.5)$$

In expressions above, $\Theta(z)$ is the Unit Step function. We have used also the following notations

$$\begin{aligned} N_1 = & s\alpha\beta\gamma \left[\gamma^3 + 2\gamma^2(\beta + \alpha - 1) + \alpha(\beta + \alpha - 1) + \gamma \left(1 + \beta^2 - 3\alpha + 2\alpha^2 \right. \right. \\ & + \beta(-2 + 3\alpha)) \left. \right] - m_Q^2 \left[\beta\alpha^2(\alpha + \beta - 1)^2 + \gamma^4(\alpha + \beta) + \gamma\alpha(\alpha + \beta - 1)^2(2\beta + \alpha) \right. \\ & + 2\gamma^3(\beta^2 + \alpha(\alpha - 1) + \beta(2\alpha - 1)) + \gamma^2(\beta^3 + \beta^2(5\alpha - 2) + \alpha(1 - 3\alpha + 2\alpha^2) + \beta(1 - 6\alpha + 6\alpha^2)) \left. \right], \\ N_2 = & \beta\alpha(\alpha + \beta - 1) + \gamma^2(\alpha + \beta) + \gamma \left[\beta^2 + \alpha(\alpha - 1) + \beta(2\alpha - 1) \right], \\ N_3 = & \gamma^2 + (\gamma + \alpha)(\beta + \alpha - 1), \quad L = \alpha + \beta + \gamma - 1, \quad L_1 = N_1/N_2^2. \end{aligned} \quad (A.6)$$

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