



Letter

Heavy four-quark mesons $bc\bar{b}\bar{c}$: Scalar particleS.S. Agaev^a, K. Azizi^{b,c,d,*}, H. Sundu^e^a Institute for Physical Problems, Baku State University, Az-1148 Baku, Azerbaijan^b Department of Physics, University of Tehran, North Karegar Avenue, Tehran 14395-547, Iran^c Department of Physics, Doğuş University, Dudullu-Ümraniye, 34775 Istanbul, Türkiye^d Department of Physics and Technical Sciences, Western Caspian University, Baku, AZ 1001, Azerbaijan^e Department of Physics Engineering, Istanbul Medeniyet University, 34700 Istanbul, Türkiye

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ABSTRACT

Parameters of the heavy four-quark scalar meson $T_{bc\bar{b}\bar{c}}$ with content $bc\bar{b}\bar{c}$ are calculated by means of the sum rule method. This structure is considered as a diquark-antidiquark state built of scalar diquark and antidiquark components. The mass and current coupling of $T_{bc\bar{b}\bar{c}}$ are evaluated in the context of the two-point sum rule approach. The full width of this tetraquark is estimated by taking into account two types of its possible strong decay channels. First class includes dissociation of $T_{bc\bar{b}\bar{c}}$ to mesons $\eta_c\eta_b$, $B_c^+B_c^-$, $B_c^{*+}B_c^{*-}$ and $B_c^+(1^3P_0)B_c^{*-}$. Another type of processes are generated by annihilations $\bar{b}b \rightarrow \bar{q}q$ of constituent b -quarks which produces the final-state charmed meson pairs D^+D^- , $D^0\bar{D}^0$, $D^{*+}D^{*-}$, and $D^{*0}\bar{D}^{*0}$. Partial width all of these decays are found using the three-point sum rule method which is required to calculate strong couplings at corresponding meson-meson-tetraquark vertices. Predictions obtained for the mass $m = (12697 \pm 90)$ MeV and width $\Gamma[T_{bc\bar{b}\bar{c}}] = (142.4 \pm 16.9)$ MeV of this state are compared with alternative results, and are useful for further experimental investigations of fully heavy resonances.

1. Introduction

Due to recent experimental achievements of different collaborations [1–3], fully heavy four-quark mesons composed of only c and b quarks became objects of intensive studies. The four X resonances observed in di- J/ψ and $J/\psi\psi'$ mass distributions by the LHCb, ATLAS, and CMS Collaborations are first candidates to such exotic mesons. These structures with the masses in the interval 6.2–7.3 GeV are presumably scalar particles $cc\bar{c}\bar{c}$ composed of four valence c quarks.

Features of X resonances were investigated in the context of different models and theoretical approaches [4–17]. The fully charmed scalar four-quark mesons in the diquark-antidiquark and molecule pictures were considered in our publications as well [18–21]. By applying the QCD two- and three-point sum rule methods, we computed masses and widths of such states. Having compared obtained predictions with experimental data, we concluded that the lightest structure $X(6200)$ is supposedly a molecule $\eta_c\eta_c$ [19], whereas $X(6600)$ maybe is the diquark-antidiquark state built of axial-vector ingredients [18]. The superposition of a tetraquark with a pseudoscalar diquark and antidiquark components and hadronic molecule $\chi_{c0}\chi_{c0}$ has parameters which agree

with parameters of the state $X(6900)$ [19,20]. The heaviest resonance $X(7300)$ can be considered as an admixture of the hadronic molecule $\chi_{c1}\chi_{c1}$ and first radial excitation of $X(6600)$ [21].

An interesting class of fully heavy tetraquarks which deserve detailed analysis are particles $bb\bar{c}\bar{c}/cc\bar{b}\bar{b}$. These exotic mesons bear two units of electric charge and under certain conditions can be strong-interaction stable states. Tetraquarks $bb\bar{c}\bar{c}/cc\bar{b}\bar{b}$ with different quantum numbers were explored in numerous publications [22–25], in which were made contradictory conclusions about their stability. The four-quark mesons $bb\bar{c}\bar{c}$ with spin-parities $J^P = 0^\pm, 1^\pm$, and 2^+ were explored in our works [26–29]. We calculated the masses of the diquark-antidiquark states with these quantum numbers and found that none of them are strong-interaction stable particles. We also evaluated widths of these states by considering their kinematically allowed decay channels.

All-heavy exotic mesons $bc\bar{b}\bar{c}$ form another group of interesting particles. These states were explored intensively in the literature as well [10,22,25,30–35], where masses of structures $bc\bar{b}\bar{c}$ with different spin-parities were calculated by applying alternative approaches. Thus, in Ref. [10] analyses were done in the context of the relativistic quark model, whereas the authors in Refs. [34,35] used the QCD

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sum rule and relativistic four-body Faddeev-Yakubovsky approaches, respectively. Predictions obtained in these works for parameters of the tetraquarks $b\bar{c}b\bar{c}$ differ from each another considerably. Thus, the mass of the scalar particle $J^{PC} = 0^{++}$ was found equal to 12.813 GeV, 12.28 – 12.46 GeV, and 10.72 GeV, respectively. The large differences between these results are evident which makes actual new detailed studies of the tetraquarks $b\bar{c}b\bar{c}$.

In present article, we consider the scalar diquark-antidiquark state $T_{b\bar{c}b\bar{c}}^-$ with the quark content $b\bar{c}b\bar{c}$ built of scalar diquark components. We are going to perform comprehensive analysis of this structure and calculate its mass and full width. To determine the mass and current coupling of $T_{b\bar{c}b\bar{c}}^-$, we make use of the two-point sum rule (SR) approach [36,37]. It turns out that this exotic meson lies above the $\eta_b\eta_c$, $B_c^+B_c^-$, $B_c^{*+}B_c^{*-}$ and $B_c^+(1^3P_0)B_c^{*-}$ thresholds and hence dissociates to these two-meson final states. Apart from that, due to $b\bar{b}$ annihilation to light quarks $T_{b\bar{c}b\bar{c}}^-$ can easily decay to D^+D^- , $D^0\bar{D}^0$, $D^{*+}D^{*-}$, and $D^{*0}\bar{D}^{*0}$ mesons: These processes are similar to ones considered in Refs. [7,8,38] in the case of structures $bb\bar{b}\bar{b}/cc\bar{c}\bar{c}$. We calculate the full width of the tetraquark $T_{b\bar{c}b\bar{c}}^-$ by taking into account these eight decay channels. Because partial widths of aforementioned modes are determined by strong couplings at corresponding tetraquark-meson-meson vertices, we estimate them by employing the three-point SR method. The sum rule computations actually give information about the strong form factor for a vertex under consideration, but these data can be employed to find an extrapolating function and fix a coupling of interest. In the case of the fall-apart processes application of the SR method is standard and straightforward. But to analyze decays of $T_{b\bar{c}b\bar{c}}^-$ generated by constituent b -quarks annihilation, we perform an intermediate step and replace $\langle b\bar{b} \rangle$ vacuum matrix element in the three-point correlation function by the gluon condensate $\langle \alpha_s G^2/\pi \rangle$. There are, in general, another channels for decays of the tetraquark $T_{b\bar{c}b\bar{c}}^-$ to conventional particles. Transformations to 4-leptons or 2-leptons+quarkonium are among such processes. But in the present work, we restrict ourselves by analysis of $T_{b\bar{c}b\bar{c}}^-$ tetraquark's strong decays.

We organize this paper in the following manner: In Sec. 2, we evaluate the mass and current coupling of the tetraquark $T_{b\bar{c}b\bar{c}}^-$. We consider the allowed fall-apart decays of $T_{b\bar{c}b\bar{c}}^-$ in Sec. 3. The partial widths of the channels D^+D^- , $D^0\bar{D}^0$, $D^{*+}D^{*-}$, and $D^{*0}\bar{D}^{*0}$ are calculated in Sec. 4. Here, we find also the full width of the scalar tetraquark $T_{b\bar{c}b\bar{c}}^-$. In Sec. 5, we compare our prediction for the mass of $T_{b\bar{c}b\bar{c}}^-$ with results available in the literature and make our brief conclusions.

2. The mass and current coupling of the scalar structure $T_{b\bar{c}b\bar{c}}^-$

The spectroscopic parameters m and Λ of the scalar diquark-antidiquark state $T_{b\bar{c}b\bar{c}}^-$ are important quantities which characterize it as four-quark meson and determine its allowed decay modes. These parameters can be evaluated using the two-point sum rule method elaborated in Refs. [36,37].

To determine the sum rules for the mass m and current coupling Λ , one needs to study the correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ J(x) J^\dagger(0) \} | 0 \rangle, \quad (1)$$

where $J(x)$ is the interpolating current for the tetraquark $T_{b\bar{c}b\bar{c}}^-$, and $\mathcal{T}$ indicates the time-ordering product of two currents.

We model $T_{b\bar{c}b\bar{c}}^-$ in the diquark-antidiquark picture as a state composed of the scalar diquarks $b^T C \gamma_5 c$ and $\bar{b} \gamma_5 \bar{c}^T$ which are most stable two-quark structures [39]. Then the current $J(x)$ takes the following form

$$J(x) = [b_a^T(x) C \gamma_5 c_b(x)] [\bar{b}_a(x) \gamma_5 \bar{c}_b^T(x)], \quad (2)$$

with a, b being the color indices and C is the charge conjugation matrix. This current describes the scalar particle with the spin-parity $J^P = 0^+$.

The SRs for the mass and current coupling of the state $T_{b\bar{c}b\bar{c}}^-$ can be extracted after computing the correlation function $\Pi(p)$ using both the physical parameters of $T_{b\bar{c}b\bar{c}}^-$ and quark-gluon degrees of freedom. The first function $\Pi^{\text{Phys}}(p)$ forms the physical side of the sum rule, whereas $\Pi^{\text{OPE}}(p)$ constitutes its QCD side.

The correlation function $\Pi^{\text{Phys}}(p)$ is given by the expression

$$\Pi^{\text{Phys}}(p) = \frac{\langle 0 | J | T_{b\bar{c}b\bar{c}}^- \rangle \langle T_{b\bar{c}b\bar{c}}^- | J^\dagger | 0 \rangle}{m^2 - p^2} + \dots \quad (3)$$

The term written down explicitly in Eq. (3) is a contribution to $\Pi^{\text{Phys}}(p)$ arising from the ground-level particle. Contributions to the correlator coming from higher resonances and continuum states are shown by dots.

To simplify $\Pi^{\text{Phys}}(p)$, we rewrite it in terms of the parameters m and Λ . For these purposes, we introduce the matrix element

$$\langle 0 | J | T_{b\bar{c}b\bar{c}}^- \rangle = \Lambda. \quad (4)$$

After simple manipulations, one gets

$$\Pi^{\text{Phys}}(p) = \frac{\Lambda^2}{m^2 - p^2} + \dots \quad (5)$$

The correlator $\Pi^{\text{Phys}}(p)$ contains a term which has the trivial Lorentz structure proportional to I , therefore $\Lambda^2/(m^2 - p^2)$ in the right-hand side of Eq. (5) is the invariant amplitude $\Pi^{\text{Phys}}(p^2)$ required for future studies.

The function $\Pi(p)$ has to be also computed using the heavy quark propagators and operator product expansion (OPE) with certain accuracy. This function forms the QCD side $\Pi^{\text{OPE}}(p)$ of the SRs and consists of the perturbative contribution, as well as contains a nonperturbative term which is proportional to $\langle \alpha_s G^2/\pi \rangle$. This is connected with the fact that heavy quark propagators do not contain light quark and mixed quark-gluon condensates. Therefore, possible additional contributions to $\Pi^{\text{OPE}}(p)$ are proportional to $\langle g_s^3 G^3 \rangle$ and $\langle \alpha_s G^2/\pi \rangle^2$, which we neglect in our following investigations.

The $\Pi^{\text{OPE}}(p)$ expressed using the b and c quarks' propagators reads

$$\begin{aligned} \Pi^{\text{OPE}}(p) = i \int d^4x e^{ipx} \text{Tr} \left[\gamma_5 \tilde{S}_c^{b'b}(-x) \gamma_5 S_b^{a'a}(-x) \right] \\ \times \text{Tr} \left[\gamma_5 \tilde{S}_b^{a'a}(x) \gamma_5 S_c^{bb'}(x) \right], \end{aligned} \quad (6)$$

where

$$\tilde{S}_Q(x) = C S_Q^T(x) C, \quad Q = b, c. \quad (7)$$

In Eq. (6) $S_{b(c)}(x)$ are propagators of the b and c -quarks [40].

The $\Pi^{\text{OPE}}(p)$ has also a simple Lorentz structure $\sim I$, and is characterized by the invariant amplitude $\Pi^{\text{OPE}}(p^2)$. The SRs for the m and Λ are obtained by equating $\Pi^{\text{OPE}}(p^2)$ and $\Pi^{\text{Phys}}(p^2)$, applying the Borel transformation and performing continuum subtraction in accordance with the assumption on quark-hadron duality [36,37]. After these operations, we find the SRs for the mass and current coupling

$$m^2 = \frac{\Pi'(M^2, s_0)}{\Pi(M^2, s_0)}, \quad (8)$$

and

$$\Lambda^2 = e^{m^2/M^2} \Pi(M^2, s_0). \quad (9)$$

Here $\Pi(M^2, s_0)$ is the amplitude $\Pi^{\text{OPE}}(p^2)$ after the Borel transformation and continuum subtraction. It is a function of the Borel and continuum subtraction parameters M^2 and s_0 . Above we used also the short-hand notation $\Pi'(M^2, s_0) = d\Pi(M^2, s_0)/d(-1/M^2)$.

The amplitude $\Pi(M^2, s_0)$ is calculated as an integral of the two-point spectral density $\rho^{\text{OPE}}(s)$

$$\Pi(M^2, s_0) = \int_{4M^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}, \quad (10)$$

where $\mathcal{M} = (m_b + m_c)$. The spectral density $\rho^{\text{OPE}}(s)$ is determined as an imaginary part of the invariant amplitude $\Pi^{\text{OPE}}(p^2)$ and is a sum of perturbative $\rho^{\text{pert.}}(s)$ and nonperturbative $\rho^{\text{Dim4}}(s)$ terms, which are given by the general expression

$$\rho(s) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma \rho(s, \alpha, \beta, \gamma), \quad (11)$$

with α , β , and γ being the Feynman parameters. The function $\rho^{\text{pert.}}(s, \alpha, \beta, \gamma)$ has the following form

$$\begin{aligned} \rho^{\text{pert.}}(s, \alpha, \beta, \gamma) = & \frac{3N^2\Theta(N)}{1024A^4B^2C^4\pi^6} \{6B^2C^2 \\ & \times (L_1 s \alpha^2 \beta^2 \gamma - A^2 m_b m_c) (A^2 m_b m_c + L_1^2 s \alpha \beta \gamma^2) \\ & - A^4 N [12CL_1^2 s \alpha^2 \beta^2 \gamma^2 + B^2 (-3L_1 N \alpha \beta \gamma + 4Cm_b m_c \\ & \times (\alpha(\beta - \gamma) + \gamma - \gamma(\beta + \gamma)))] \}, \end{aligned} \quad (12)$$

where $\Theta(z)$ is the Unit Step function. Here

$$N = -C [s\alpha\beta\gamma L_1 + D(m_c^2 L_2 - m_b^2(\alpha + \beta))] / D^2, \quad (13)$$

and

$$\begin{aligned} A &= L_4 C + \alpha^2(\beta + \gamma), \quad B = L_2 C + \gamma^2(\beta + \alpha), \\ C &= \alpha\beta + \alpha\gamma + \beta\gamma, \quad D = \gamma\beta L_4 + \alpha^2(\beta + \gamma) \\ &+ \alpha(\beta^2 + \gamma(\gamma - 1) + \beta(2\gamma - 1)). \end{aligned} \quad (14)$$

We also use the notations

$$\begin{aligned} L_1 &= \alpha + \beta + \gamma - 1, \quad L_2 = \alpha + \beta - 1, \\ L_3 &= \alpha + \gamma - 1, \quad L_4 = \beta + \gamma - 1. \end{aligned} \quad (15)$$

An explicit formula for $\rho^{\text{Dim4}}(s, \alpha, \beta, \gamma)$ is rather cumbersome and is not provided here.

The SRs for the mass m and current coupling Λ depend on the masses of the heavy quarks and on the gluon condensate

$$\begin{aligned} m_b &= 4.18_{-0.02}^{+0.03} \text{ GeV}, \quad m_c = (1.27 \pm 0.02) \text{ GeV}, \\ \langle \alpha_s G^2 / \pi \rangle &= (0.012 \pm 0.004) \text{ GeV}^4. \end{aligned} \quad (16)$$

There are also in Eqs. (8) and (9) the parameters M^2 and s_0 a choice of which must satisfy constraints typical for the sum rule method. In other words, they should lead to prevalence of the pole contribution (PC)

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \quad (17)$$

by meeting the restriction $\text{PC} \geq 0.5$. The upper limit of M^2 is extracted from this constraint. The convergence of OPE determines the lower bound for the Borel parameter. In our investigation there is a nonperturbative term with dimension 4, therefore we fix a minimal value of M^2 by restricting its contribution by $\pm(5 - 15)\%$ of $\Pi(M^2, s_0)$. This constraint also leads to dominance of the perturbative contribution in $\Pi(M^2, s_0)$. Another important constraint is a stability of obtained quantities on the parameter M^2 .

Our numerical analysis demonstrates that these conditions are satisfied by the working windows

$$M^2 \in [12, 14] \text{ GeV}^2, \quad s_0 \in [180, 185] \text{ GeV}^2. \quad (18)$$

Really, at $M^2 = 14 \text{ GeV}^2$ on the average in s_0 the pole contribution is equal to $\text{PC} \approx 0.51$, whereas at $M^2 = 12 \text{ GeV}^2$ it amounts to ≈ 0.65 . The nonperturbative contribution is negative and at $M^2 = 12 \text{ GeV}^2$ constitutes only 1% of the whole result. In Fig. 1, we depict the pole contribution PC as a function of the Borel parameter at the fixed s_0 . As is seen, it exceeds 0.5 at all values of M^2 and s_0 .

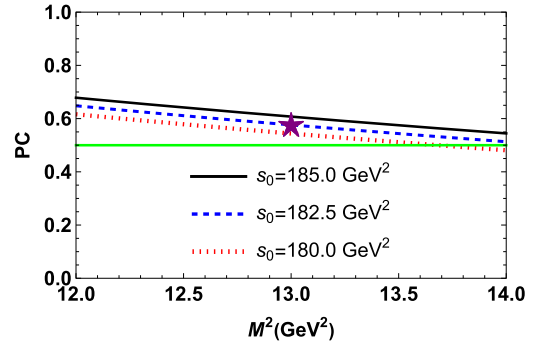


Fig. 1. The pole contribution PC as a function of M^2 at fixed s_0 . The red star marks the point $M^2 = 13 \text{ GeV}^2$ and $s_0 = 182.5 \text{ GeV}^2$.

The mass m and current coupling Λ are calculated as mean values of these parameters over the regions Eq. (18) and are equal to

$$\begin{aligned} m &= (12697 \pm 90) \text{ MeV}, \\ \Lambda &= (1.50 \pm 0.17) \text{ GeV}^5. \end{aligned} \quad (19)$$

The predictions Eq. (19) effectively correspond to SR results at the point $M^2 = 13 \text{ GeV}^2$ and $s_0 = 182.5 \text{ GeV}^2$, where $\text{PC} \approx 0.58$. This fact guarantees the dominance of PC in the extracted results, and demonstrates ground-level nature of the tetraquark $T_{bc\bar{b}\bar{c}}$ in its class. Ambiguities in Eq. (19) are mainly generated by choices of the parameters M^2 and s_0 . In the case of the mass m they form only $\pm 0.07\%$ of the obtained result which implies very high stability of performed analysis. This fact can be explained by the SR Eq. (8) defined as a ratio of two correlation functions. As a result, variations in correlators due to M^2 , s_0 and m_b, m_c are damped in m which stabilizes numerical output. In the case of Λ ambiguities are equal to $\pm 11\%$ remaining within acceptable limits of the sum rule analysis. In Fig. 2, we depict m as a function of M^2 and s_0 .

3. Decays of $T_{bc\bar{b}\bar{c}}$: fall-apart processes

The mass m of the scalar tetraquark $T_{bc\bar{b}\bar{c}}$ allows us to fix its kinematically allowed decay modes. It is clear that a tetraquark with a content $bc\bar{b}\bar{c}$ may dissociate to conventional mesons $\bar{c}b + \bar{b}c$ and $\bar{c}c + \bar{b}b$ with appropriate quantum numbers provided its mass exceeds the mass of the final two-meson states. These decays are fall apart processes in which constituent quarks are distributed between final-state mesons.

Information on masses of the ordinary $\bar{c}b(b\bar{c})$ and $\bar{c}c(b\bar{b})$ mesons can be found in Ref. [41]. It is worth noting that only the mass of the pseudoscalar ground-level and radially excited mesons $B_c^\pm$ and $B_c^\pm(2S)$ are measured experimentally. Parameters of particles with the same content but other quantum numbers were calculated in the context of different methods. In the present work, we employ predictions obtained in Ref. [42]. The quarkonia $\bar{c}c(b\bar{b})$ were investigated in detailed form and their masses are known with a rather high precision. Corresponding information about parameters of the mesons which will be considered in this and next section is collected in Table 1.

It is not difficult to see that dissociation to pairs of $\eta_b \eta_c$, $B_c^+ B_c^-$, $B_c^{*+} B_c^{*-}$ and $B_c^+(1^3P_0) B_c^{*-}$ mesons are kinematically allowed decay channels of the tetraquark $T_{bc\bar{b}\bar{c}}$. In this section, we study these modes and calculate their partial widths.

3.1. $T_{bc\bar{b}\bar{c}} \rightarrow \eta_b \eta_c$

We begin from investigation of the decay $T_{bc\bar{b}\bar{c}} \rightarrow \eta_b \eta_c$. The width of this process, apart from parameters of particles involved into this decay, depends also on the strong coupling g_1 at the vertex $T_{bc\bar{b}\bar{c}} \eta_b \eta_c$. To determine g_1 , we study the QCD three-point correlation function

$$\Pi_1(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J^{\eta_b}(y) \}$$

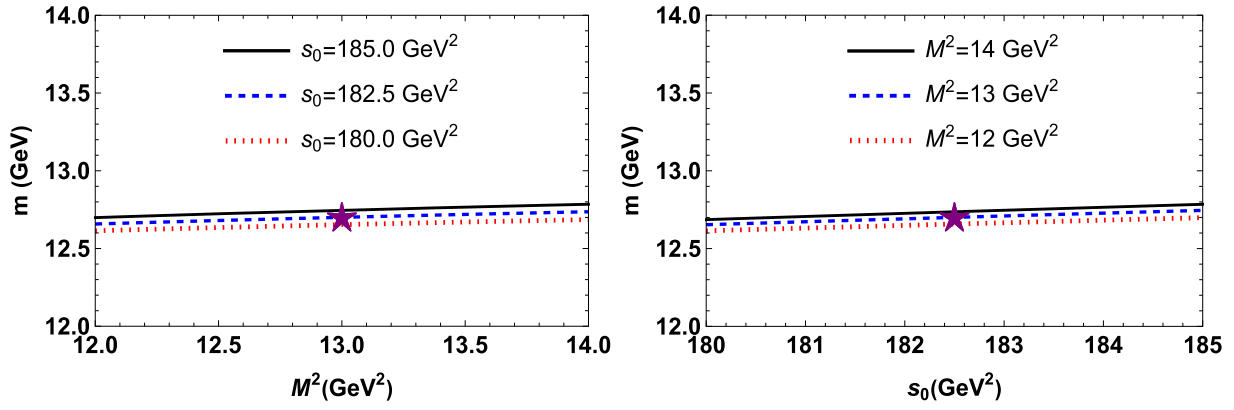


Fig. 2. Mass m of the tetraquark $T_{(bc)}$ as a function of the Borel M^2 (left panel), and continuum threshold s_0 parameters (right panel).

Table 1

Physical parameters of the conventional mesons, which have been employed in numerical computations.

Quantity	Value (in MeV units)
m_{η_b}	9398.7 ± 2.0
m_{η_c}	2983.9 ± 0.4
m_{B_c}	$6274.47 \pm 0.27 \pm 0.17$
$m_{B_c^*}$	6338
$m_{B_{c1}}$	6706
m_D	1869.5 ± 0.05
m_{D^0}	1864.84 ± 0.05
m_{D^*}	2010.26 ± 0.05
$m_{D^{*0}}$	2006.85 ± 0.05
f_{η_b}	724
f_{η_c}	421 ± 35
f_{B_c}	371 ± 37
$f_{B_c^*}$	471
$f_{B_{c1}}$	236 ± 17
f_D	211.9 ± 1.1
f_{D^*}	252.2 ± 22.66

$$\times J^{\eta_c}(0) J^{\dagger}(x) |0\rangle, \quad (20)$$

where

$$\begin{aligned} J^{\eta_b}(x) &= \bar{b}_i(x) i \gamma_5 b_i(x), \\ J^{\eta_c}(x) &= \bar{c}_j(x) i \gamma_5 c_j(x), \end{aligned} \quad (21)$$

are the interpolating currents of the pseudoscalar quarkonia η_b and η_c , respectively.

By studying the correlation function $\Pi_1(p, p')$ it is possible to derive the SR for the strong form factor $g_1(q^2)$, which at the mass shell $q^2 = m_{\eta_c}^2$ becomes equal to g_1 . To determine the sum rule for the form factor $g_1(q^2)$, we employ well-known recipes of the method, and first write down $\Pi_1(p, p')$ in terms of masses and current coupling (decay constants) of the tetraquark $T_{bc\bar{c}}$ and quarkonia η_b and η_c . The correlation function $\Pi_1^{\text{Phys}}(p, p')$ calculated by this way forms the physical side of the sum rule and is given by the expression

$$\begin{aligned} \Pi_1^{\text{Phys}}(p, p') &= \frac{\langle 0 | J^{\eta_b} | \eta_b(p') \rangle \langle 0 | J^{\eta_c} | \eta_c(q) \rangle}{p'^2 - m_{\eta_b}^2} \frac{q^2 - m_{\eta_c}^2}{q^2 - m_{\eta_c}^2} \\ &\times \langle \eta_b(p') \eta_c(q) | T_{bc\bar{c}} | J^{\dagger} | 0 \rangle \frac{\langle T_{bc\bar{c}}(p) | J^{\dagger} | 0 \rangle}{p^2 - m^2} + \dots, \end{aligned} \quad (22)$$

where m_{η_b} and m_{η_c} are the masses of η_b and η_c . The correlation function $\Pi_1^{\text{Phys}}(p, p')$ is obtained after separating a contribution of the ground-level particles from ones due to higher resonances and continuum states denoted above by the dots.

We continue to simplify Eq. (22) by introducing the matrix elements

$$\begin{aligned} \langle 0 | J^{\eta_b} | \eta_b(p') \rangle &= \frac{f_{\eta_b} m_{\eta_b}^2}{2m_b}, \\ \langle 0 | J^{\eta_c} | \eta_c(q) \rangle &= \frac{f_{\eta_c} m_{\eta_c}^2}{2m_c}, \end{aligned} \quad (23)$$

with f_{η_b} and f_{η_c} being the decay constants of the mesons η_b and η_c [43], respectively. We model the vertex $T_{bc\bar{c}} \eta_b \eta_c$ by means of the formula

$$\langle \eta_b(p') \eta_c(q) | T_{bc\bar{c}} | p \rangle = g_1(q^2) p \cdot p'. \quad (24)$$

Then, the correlation function $\Pi_1^{\text{Phys}}(p, p')$ takes the following form

$$\begin{aligned} \Pi_1^{\text{Phys}}(p, p') &= g_1(q^2) \frac{\Lambda f_{\eta_b} m_{\eta_b}^2 f_{\eta_c} m_{\eta_c}^2}{8m_b m_c (p^2 - m^2)} \\ &\times \frac{m^2 + m_{\eta_b}^2 - q^2}{(p'^2 - m_{\eta_b}^2)(q^2 - m_{\eta_c}^2)} + \dots \end{aligned} \quad (25)$$

As is seen, $\Pi_1^{\text{Phys}}(p, p')$ has a simple Lorentz structure proportional to I , therefore we accept the right-hand side of Eq. (25) as the invariant amplitude and denote it by $\Pi_1^{\text{Phys}}(p^2, p'^2, q^2)$.

The same correlator $\Pi_1(p, p')$ computed in terms of the heavy quark propagators is

$$\begin{aligned} \Pi_1^{\text{OPE}}(p, p') &= i^4 \int d^4 x d^4 y e^{ip'y} e^{-ipx} \text{Tr} \left[\gamma_5 \tilde{S}_b^{ia}(y-x) \right. \\ &\times \gamma_5 \tilde{S}_b^{ai}(x-y) \gamma_5 S_c^{bj}(x) \gamma_5 S_c^{jb}(-x) \left. \right]. \end{aligned} \quad (26)$$

We stand the function $\Pi_1^{\text{OPE}}(p^2, p'^2, q^2)$ for the invariant amplitude that correspond in $\Pi_1^{\text{OPE}}(p, p')$ to term $\sim I$, and employ it to find the SR for the form factor $g_1(q^2)$

$$\begin{aligned} g_1(q^2) &= \frac{8m_b m_c}{\Lambda f_{\eta_b} m_{\eta_b}^2 f_{\eta_c} m_{\eta_c}^2} \frac{q^2 - m_{\eta_c}^2}{m^2 + m_{\eta_b}^2 - q^2} \\ &\times e^{m^2/M_1^2} e^{m_{\eta_b}^2/M_2^2} \Pi_1(M^2, s_0, q^2), \end{aligned} \quad (27)$$

where

$$\begin{aligned} \Pi_1(M^2, s_0, q^2) &= \int_{4M^2}^{s_0} ds \int_{4m_b^2}^{s_0'} ds' \rho_1(s, s', q^2) \\ &\times e^{-s/M_1^2} e^{-s'/M_2^2}, \end{aligned} \quad (28)$$

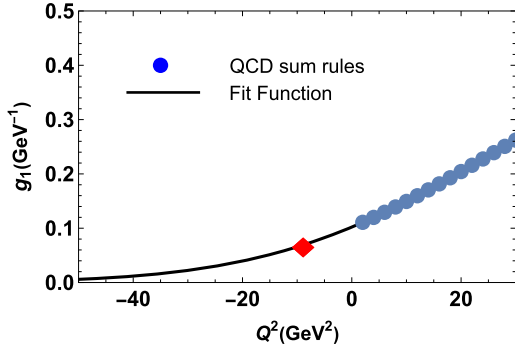


Fig. 3. QCD data and fit function for the form factor $g_1(Q^2)$. The diamond fix the point $Q^2 = -m_{\eta_c}^2$ where g_1 has been evaluated.

is the function $\Pi_1^{\text{OPE}}(p^2, p'^2, q^2)$ after the double Borel transformations over the variables $-p^2$, $-p'^2$ and corresponding continuum subtractions procedures. It is written using the spectral density $\rho_1(s, s', q^2)$.

The sum rule in Eq. (27) depends on the Borel $M^2 = (M_1^2, M_2^2)$ and continuum threshold parameters $s_0 = (s_0, s'_0)$. To perform numerical analysis, in the $T_{bc\bar{b}c}$ channel we employ M_1^2 and s_0 given in Eq. (18). The parameters (M_2^2, s'_0) in the η_b channel are varied inside of the borders

$$M_2^2 \in [9, 11] \text{ GeV}^2, \quad s'_0 \in [95, 99] \text{ GeV}^2. \quad (29)$$

It should be noted that $\sqrt{s'_0}$ is limited by the mass 9.999 GeV of the first radially excited $\eta_b(2S)$ meson.

The credible results for the form factor $g_1(q^2)$ in the sum rule framework can be obtained in the Euclidean region $q^2 < 0$. At the same time, the coupling g_1 should be fixed at the mass shell $q^2 = m_{\eta_c}^2$. To solve this problem, it is convenient to introduce a variable $Q^2 = -q^2$ and employ $g_1(Q^2)$ for the new function. Then, we use an extrapolating function $G_1(Q^2)$ which for $Q^2 > 0$ gives values coinciding with the SR results, but can be extended to the region of negative $Q^2 < 0$. To this end, we use the function $G_i(Q^2)$

$$G_i(Q^2) = G_i^0 \exp \left[a_i^1 \frac{Q^2}{m^2} + a_i^2 \left(\frac{Q^2}{m^2} \right)^2 \right], \quad (30)$$

where G_i^0 , a_i^1 , and a_i^2 are fitting parameters.

In this study, SR calculations cover the region $Q^2 = 2 - 30 \text{ GeV}^2$. Results of numerical calculations are shown in Fig. 3. It is not difficult to find that the function $G_1(Q^2)$ with parameters $G_1^0 = 0.10 \text{ GeV}^{-1}$, $a_1^1 = 7.03$, and $a_1^2 = -9.46$ leads to reasonable agreement with the SR data: This function is also plotted in Fig. 3.

As a result, we get for g_1

$$g_1 \equiv G_1(-m_{\eta_c}^2) = (6.9 \pm 0.9) \times 10^{-2} \text{ GeV}^{-1}. \quad (31)$$

The width of the decay $T_{bc\bar{b}c} \rightarrow \eta_b \eta_c$ can be calculated by means of the expression

$$\Gamma[T_{bc\bar{b}c} \rightarrow \eta_b \eta_c] = g_1^2 \frac{m_{\eta_b}^2 \lambda_1}{8\pi} \left(1 + \frac{\lambda_1^2}{m_{\eta_b}^2} \right), \quad (32)$$

where $\lambda_1 = \lambda(m, m_{\eta_b}, m_{\eta_c})$, and

$$\lambda(x, y, z) = \frac{\sqrt{x^4 + y^4 + z^4 - 2(x^2 y^2 + x^2 z^2 + y^2 z^2)}}{2x}. \quad (33)$$

Finally, we find

$$\Gamma[T_{bc\bar{b}c} \rightarrow \eta_b \eta_c] = (32.4 \pm 8.3) \text{ MeV}. \quad (34)$$

3.2. $T_{bc\bar{b}c} \rightarrow B_c^+ B_c^-$

The partial width of the decay $T_{bc\bar{b}c} \rightarrow B_c^+ B_c^-$ is determined by the strong coupling g_2 at the vertex $T_{bc\bar{b}c} B_c^+ B_c^-$. In the framework of the QCD SR method the form factor $g_2(q^2)$ is calculated using the three-point correlation function

$$\begin{aligned} \Pi_2(p, p') &= i^2 \int d^4 x d^4 y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J^{B_c^+}(y) \\ &\times J^{B_c^-}(0) J^\dagger(x) \} | 0 \rangle, \end{aligned} \quad (35)$$

where $J^{B_c^+}(y)$ and $J^{B_c^-}(0)$ are the interpolating currents for the mesons B_c^+ and B_c^-

$$\begin{aligned} J^{B_c^+}(x) &= \bar{b}_i(x) i \gamma_5 c_i(x), \\ J^{B_c^-}(x) &= \bar{c}_j(x) i \gamma_5 b_j(x), \end{aligned} \quad (36)$$

respectively.

To determine the physical side of the SR, we use the matrix elements

$$\langle 0 | J^{B_c^\pm} | B_c^\pm \rangle = \frac{f_{B_c} m_{B_c}^2}{m_b + m_c}, \quad (37)$$

and

$$\langle B_c^+(p') B_c^-(q) | T_{bc\bar{b}c} \rangle = g_2(q^2) p \cdot p', \quad (38)$$

where m_{B_c} and f_{B_c} are the mass and decay constant of the mesons $B_c^\pm$ [41,44]. Then in terms of these parameters the correlator $\Pi_2(p, p')$ takes the form

$$\begin{aligned} \Pi_2^{\text{Phys}}(p, p') &= \frac{g_2(q^2) \Lambda f_{B_c}^2 m_{B_c}^4}{(m_b + m_c)^2 (p^2 - m^2) (p'^2 - m_{B_c}^2)} \\ &\times \frac{m^2 + m_{B_c}^2 - q^2}{2(q^2 - m_{B_c}^2)} + \dots \end{aligned} \quad (39)$$

The QCD side of the sum rule for the coupling $g_2(q^2)$ is given by the expression

$$\begin{aligned} \Pi_2^{\text{OPE}}(p, p') &= -i^4 \int d^4 x d^4 y e^{ip'y} e^{-ipx} \text{Tr} \left[\gamma_5 S_c^{ib}(y-x) \right. \\ &\times \gamma_5 \tilde{S}_b^{ja}(-x) \gamma_5 \tilde{S}_c^{bj}(x) \gamma_5 S_b^{ai}(x-y) \left. \right]. \end{aligned} \quad (40)$$

In this decay the correlators $\Pi_2^{\text{Phys}}(p, p')$ and $\Pi_2^{\text{OPE}}(p, p')$ have simple Lorentz organizations. Having denoted by $\Pi_2^{\text{Phys}}(p^2, p'^2, q^2)$ and $\Pi_2^{\text{OPE}}(p^2, p'^2, q^2)$ relevant invariant amplitudes, we derive the following sum rule

$$\begin{aligned} g_2(q^2) &= \frac{2(m_b + m_c)^2}{\Lambda f_{B_c}^2 m_{B_c}^4} \frac{q^2 - m_{B_c}^2}{m^2 + m_{B_c}^2 - q^2} \\ &\times e^{m^2/M_1^2} e^{m_{B_c}^2/M_2^2} \Pi_2(M^2, s_0, q^2). \end{aligned} \quad (41)$$

Here, $\Pi_2(M^2, s_0, q^2)$ is the Borel transformed and subtracted amplitude $\Pi_2^{\text{OPE}}(p^2, p'^2, q^2)$

$$\begin{aligned} \Pi_2(M^2, s_0, q^2) &= \int_{4M^2}^{s_0} ds \int_{M^2}^{s'_0} ds' \rho_2(s, s', q^2) \\ &\times e^{-s/M_1^2} e^{-s'/M_2^2}. \end{aligned} \quad (42)$$

The remaining operations are the same as ones explained above. Therefore, we present only the windows for M_2^2 , and s'_0 in the B_c^+ meson channel

$$M_2^2 \in [6.5, 7.5] \text{ GeV}^2, \quad s'_0 \in [45, 47] \text{ GeV}^2. \quad (43)$$

The extrapolating function $\mathcal{G}_2(Q^2)$ has the parameters: $\mathcal{G}_2^0 = 0.17 \text{ GeV}^{-1}$, $a_2^1 = 1.97$, and $a_2^2 = -1.41$. Then, the strong coupling g_2 becomes equal to

$$g_2 \equiv \mathcal{G}_2(-m_{B_c}^2) = (1.0 \pm 0.2) \times 10^{-1} \text{ GeV}^{-1}. \quad (44)$$

Having used the strong coupling g_2 , and Eq. (32) with evident replacements $m_{\eta_b} \rightarrow m_{B_c}$ and $\lambda_1 \rightarrow \lambda_2 = \lambda(m, m_{B_c}, m_{B_c})$, we find

$$\Gamma[T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^+ B_c^-] = (33.9 \pm 10.1) \text{ MeV}. \quad (45)$$

3.3. $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^{*+} B_c^{*-}$

The next decay which will be considered in this subsection is the process $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^{*+} B_c^{*-}$. Treatment of this channel differs from previous modes by only some technical details. Thus the correlation function necessary to be analyzed is

$$\begin{aligned} \Pi_{\mu\nu}(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J_\mu^{B_c^{*+}}(y) \\ \times J_\nu^{B_c^{*-}}(0) J^\dagger(x) \} | 0 \rangle. \end{aligned} \quad (46)$$

Here, $J_\mu^{B_c^{*+}}(y)$ and $J_\nu^{B_c^{*-}}(0)$ are the interpolating currents of the vector mesons B_c^{*+} and B_c^{*-} , respectively. They are given by the formulas

$$\begin{aligned} J_\mu^{B_c^{*+}}(x) &= \bar{b}_i(x) \gamma_\mu c_i(x), \\ J_\nu^{B_c^{*-}}(x) &= \bar{c}_j(x) \gamma_\nu b_j(x). \end{aligned} \quad (47)$$

The physical side of the SR for the coupling $g_3(q^2)$ which corresponds to the strong vertex $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} B_c^{*+} B_c^{*-}$ is

$$\begin{aligned} \Pi_{\mu\nu}^{\text{Phys}}(p, p') &= \frac{\langle 0 | J_\mu^{B_c^{*+}} | B_c^{*+}(p') \rangle \langle 0 | J_\nu^{B_c^{*-}} | B_c^{*-}(q) \rangle}{p'^2 - m_{B_c^*}^2} \frac{q^2 - m_{B_c^*}^2}{q^2 - m_{B_c^*}^2} \\ &\times \langle B_c^{*+}(p') B_c^{*-}(q) | T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) | J^\dagger | 0 \rangle \frac{\langle T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) | J^\dagger | 0 \rangle}{p^2 - m^2} + \dots, \end{aligned} \quad (48)$$

where $m_{B_c^*}$ is the mass of the mesons $B_c^{*\pm}$. For following studies it is convenient to introduce the matrix elements

$$\begin{aligned} \langle 0 | J_\mu^{B_c^{*+}} | B_c^{*+}(p') \rangle &= f_{B_c^*} m_{B_c^*} \varepsilon_\mu(p') \\ \langle 0 | J_\nu^{B_c^{*-}} | B_c^{*-}(q) \rangle &= f_{B_c^*} m_{B_c^*} \varepsilon_\nu(q), \end{aligned} \quad (49)$$

with $f_{B_c^*}$ and $\varepsilon(p')$, $\varepsilon(q)$ being the decay constant and polarization vectors of B_c^{*+} and B_c^{*-} . We model the vertex $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} B_c^{*+} B_c^{*-}$ in the following form

$$\begin{aligned} \langle B_c^{*+}(p') B_c^{*-}(q) | T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) \rangle &= g_3(q^2) [q \cdot p' \\ &\times \varepsilon^*(p') \cdot \varepsilon^*(q) - q \cdot \varepsilon^*(p') \cdot \varepsilon^*(q)]. \end{aligned} \quad (50)$$

Then the $\Pi_{\mu\nu}(p, p')$ in terms of physical parameters of involved particles has the expression

$$\begin{aligned} \Pi_{\mu\nu}^{\text{Phys}}(p, p') &= \frac{g_3(q^2) \Lambda f_{B_c^*}^2 m_{B_c^*}^2}{(p^2 - m^2) (p'^2 - m_{B_c^*}^2) (q^2 - m_{B_c^*}^2)} \\ &\times \left[\frac{1}{2} (m^2 - m_{B_c^*}^2 - q^2) g_{\mu\nu} - q_\mu p'_\nu \right] + \dots. \end{aligned} \quad (51)$$

The correlator $\Pi_{\mu\nu}(p, p')$ obtained using the heavy quark propagators reads

$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(p, p') &= i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \text{Tr} [\gamma_\mu S_c^{ib}(y-x) \\ &\times \gamma_5 \tilde{S}_b^{ja}(-x) \gamma_\nu \tilde{S}_c^{bj}(x) \gamma_5 S_b^{ai}(x-y)]. \end{aligned} \quad (52)$$

The correlator $\Pi_{\mu\nu}(p, p')$ in both cases contains the two Lorentz structures $g_{\mu\nu}$ and $q_\mu p'_\nu$. We use invariant amplitudes that correspond to structures proportional to $g_{\mu\nu}$ and derive a required sum rule.

Numerical analysis has been carried out using the following input parameters: For the Borel and continuum subtraction parameters in the B_c^{*+} channel we have used

$$M_2^2 \in [6.5, 7.5] \text{ GeV}^2, \quad s_0' \in [50, 51] \text{ GeV}^2. \quad (53)$$

The decay constant $f_{B_c^*}$ of the mesons B_c^* has been chosen equal to 471 MeV [45]. The extrapolating function $\mathcal{G}_3(Q^2)$ has the parameters: $\mathcal{G}_3^0 = 0.22 \text{ GeV}^{-1}$, $a_3^1 = 2.65$, and $a_3^2 = -3.58$. Then, the strong coupling g_3 amounts to

$$|g_3| \equiv \mathcal{G}_3(-m_{B_c^*}^2) = (9.8 \pm 1.3) \times 10^{-2} \text{ GeV}^{-1}. \quad (54)$$

The partial width of the decay $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^{*+} B_c^{*-}$ can be found by means of the formula

$$\Gamma[T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^{*+} B_c^{*-}] = \frac{g_3^2 \lambda_3}{16\pi} \left(m^2 - 4m_{B_c^*}^2 + 6 \frac{m_{B_c^*}^4}{m^2} \right), \quad (55)$$

where $\lambda_3 = \lambda(m, m_{B_c^*}, m_{B_c^*})$. Numerical computations of the partial decay width of this channel give

$$\Gamma[T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^{*+} B_c^{*-}] = (21.1 \pm 5.8) \text{ MeV}. \quad (56)$$

3.4. $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^+(1^3P_0) B_c^{*-}$

Investigation of the decay $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^+(1^3P_0) B_c^{*-}$ has been performed in accordance with the general scheme explained above. The correlation function which should be considered in this case is given by the expression

$$\begin{aligned} \Pi_\mu(p, p') &= i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J^{B_{c1}}(y) \\ &\times J_\mu^{B_c^{*-}}(0) J^\dagger(x) \} | 0 \rangle, \end{aligned} \quad (57)$$

with $J^{B_{c1}}(y)$ being the interpolating current of the scalar meson $B_c^+(1^3P_0)$

$$J^{B_{c1}}(y) = \bar{b}_i(x) c_i(x). \quad (58)$$

The correlator $\Pi_\mu(p, p')$ in terms of the particles' parameters takes the form

$$\begin{aligned} \Pi_\mu^{\text{Phys}}(p, p') &= \frac{g_4(q^2) \Lambda f_{B_c^*} m_{B_c^*} f_{B_{c1}} m_{B_{c1}}}{(p^2 - m^2) (p'^2 - m_{B_{c1}}^2) (q^2 - m_{B_c^*}^2)} \\ &\times \left[\frac{m^2 - m_{B_{c1}}^2 - q^2}{2m_{B_c^*}^2} p_\mu - \frac{m^2 - m_{B_{c1}}^2 + q^2}{2m_{B_c^*}^2} p'_\mu \right] + \dots, \end{aligned} \quad (59)$$

where $m_{B_{c1}}$ and $f_{B_{c1}}$ are the mass and decay constant of $B_c^+(1^3P_0)$ [42,44]. The function $\Pi_\mu^{\text{Phys}}(p, p')$ is obtained by using the new matrix elements

$$\langle 0 | J^{B_{c1}} | B_c^+(1^3P_0)(p') \rangle = m_{B_{c1}} f_{B_{c1}}, \quad (60)$$

and

$$\langle B_c^+(1^3P_0)(p') B_c^{*-}(q) | T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) \rangle = g_4(q^2) \varepsilon^*(q) \cdot p. \quad (61)$$

The correlation function $\Pi_\mu(p, p')$ calculated using the heavy quark propagators is

$$\begin{aligned} \Pi_{\mu}^{\text{OPE}}(p, p') &= i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \text{Tr} \left[S_c^{ib}(y-x) \right. \\ &\quad \left. \times \gamma_5 \tilde{S}_b^{ja}(-x) \gamma_{\mu} \tilde{S}_c^{bj}(x) \gamma_5 S_b^{ai}(x-y) \right]. \end{aligned} \quad (62)$$

Having omitted further details, we present final results for the parameters of this process:

$$|g_4| \equiv G_4(-m_{B_c^*}^2) = (12.2 \pm 2.5), \quad (63)$$

and

$$\Gamma[T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow B_c^+(1^3 P_0) B_c^{*-}] = (25.4 \pm 8.1) \text{ MeV}. \quad (64)$$

4. Decays to charmed mesons

This section is devoted to decays of $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}$ to charmed mesons which become possible due to reorganization of this particle's quark content. Thus, because $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}$ contains a pair of $\bar{b}b$ (and $\bar{c}c$) quarks, they can annihilate and produce light quarks $q\bar{q}$ which later generate conventional mesons. We are going to consider processes in which $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}$ transforms to mesons $D^+ D^-$, $D^0 \bar{D}^0$, $D^{*+} D^{*-}$, and $D^{*0} \bar{D}^{*0}$. Effectively these channels are realized due to annihilation of $\bar{b}b$ quarks with subsequent creation of $u\bar{u}$ or $d\bar{d}$ pairs. In the context of the SR method these decays can be considered using the three-point correlation functions and replacing the heavy quark condensate $\langle \bar{b}b \rangle$ by the gluon condensate $\langle \alpha_s G^2/\pi \rangle$.

4.1. $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow D^0 \bar{D}^0$ and $D^+ D^-$

Let us analyze in a detailed form the process $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow D^0 \bar{D}^0$. In the case under consideration the coupling G_1 , which describes strong interaction at the $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} D^0 \bar{D}^0$ vertex, can be extracted from the three-point correlation function

$$\begin{aligned} \tilde{\Pi}(p, p') &= i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J^{D^0}(y) \\ &\quad \times J^{\bar{D}^0}(0) J^{\dagger}(x) \} | 0 \rangle, \end{aligned} \quad (65)$$

where the interpolating currents $J^{D^0}(y)$ and $J^{\bar{D}^0}(0)$ for the mesons $D^0 = c\bar{u}$ and $\bar{D}^0 = \bar{c}u$ have the following forms

$$J^{D^0}(x) = \bar{u}_j(x) i\gamma_5 c_j(x), \quad J^{\bar{D}^0}(x) = \bar{c}_i(x) i\gamma_5 u_i(x). \quad (66)$$

In accordance with the sum rule approach, we have to express the function $\Pi(p, p')$ in terms of involved particles' parameters. By this way, we determine the physical side of the sum rule. To this end, we write down the $\Pi(p, p')$ in the following form

$$\begin{aligned} \tilde{\Pi}^{\text{Phys}}(p, p') &= \frac{\langle 0 | J^{D^0} | D^0(p') \rangle \langle 0 | J^{\bar{D}^0} | \bar{D}^0(q) \rangle}{p'^2 - m_{D^0}^2} \frac{1}{q^2 - m_{D^0}^2} \\ &\quad \times \langle D^0(p') \bar{D}^0(q) | T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) | J^{\dagger} | 0 \rangle \frac{1}{p^2 - m^2} + \dots, \end{aligned} \quad (67)$$

where m_{D^0} is the D^0 and $\bar{D}^0$ mesons' mass.

To simplify $\tilde{\Pi}^{\text{Phys}}(p, p')$, we express the matrix elements in Eq. (67), using the masses and current couplings (decay constants) of particles. The matrix element of the pseudoscalar D mesons is determined by the formula

$$\langle 0 | J^{D^0} | D^0 \rangle = \frac{f_D m_{D^0}^2}{m_c}, \quad (68)$$

with f_D being the decay constant of D^0 and $\bar{D}^0$ [46]. The matrix element of the meson $\bar{D}^0$ is given by the same expression.

The vertex $\langle D^0(p') \bar{D}^0(q) | T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) \rangle$ is modeled by the formula

$$\langle D^0(p') \bar{D}^0(q) | T_{\text{bc}\bar{\text{b}}\bar{\text{c}}}(p) \rangle = G_1(q^2) p \cdot p'. \quad (69)$$

Here, $G_1(q^2)$ is the strong form factor which at the mass shell of the $\bar{D}^0$ meson, i.e., at $q^2 = m_{D^0}^2$ fixes the strong coupling G_1 .

Then, it is easy to recast $\tilde{\Pi}^{\text{Phys}}(p, p')$ into simpler form

$$\begin{aligned} \tilde{\Pi}^{\text{Phys}}(p, p') &= G_1(q^2) \frac{\Lambda f_D^2 m_D^4}{2m_c^2 (p^2 - m^2) (p'^2 - m_{D^0}^2)} \\ &\quad \times \frac{(m^2 + m_{D^0}^2 - q^2)}{(q^2 - m_{D^0}^2)} + \dots \end{aligned} \quad (70)$$

Because $\tilde{\Pi}^{\text{Phys}}(p, p')$ has trivial Lorentz organization, the whole expression in the right hand side of Eq. (70) is the invariant amplitude $\tilde{\Pi}^{\text{Phys}}(p^2, p'^2, q^2)$ which can be applied to derive the form factor $G_1(q^2)$.

The first step in the current analysis does not differ from ones explained in the previous section. The second component which is necessary to find SR for $G_1(q^2)$ is the correlator Eq. (65) computed using the quark propagators, which reads

$$\begin{aligned} \tilde{\Pi}^{\text{OPE}}(p, p') &= \int d^4x d^4y e^{ip'y} e^{-ipx} \langle \bar{b}b \rangle \\ &\quad \times \text{Tr} [\gamma_5 S_c^{ja}(y-x) S_c^{ai}(x-y) \gamma_5 S_u^{ij}(-y)], \end{aligned} \quad (71)$$

where $S_u(x)$ is the u quark propagator [40].

The function $\tilde{\Pi}^{\text{OPE}}(p, p')$ depends on three quark propagators and vacuum condensate $\langle \bar{b}b \rangle$ of b quarks. The function $\tilde{\Pi}^{\text{OPE}}(p, p')$ differs from a standard correlator. Indeed, the latter in the case, for example, of the decay $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow \eta_b \eta_c$ contains four propagators $S_{b(c)}(x)$. The reason is that to calculate $\tilde{\Pi}^{\text{OPE}}(p, p')$ we contract heavy and light quark fields and, as a result, $\bar{b}b$ quarks establish a heavy quark condensate $\langle \bar{b}b \rangle$, because mesons $D^0 \bar{D}^0$ do not contain b quarks.

Using the relation between different condensates

$$m_b \langle \bar{b}b \rangle = -\frac{1}{12\pi} \langle \frac{\alpha_s G^2}{\pi} \rangle, \quad (72)$$

we get

$$\begin{aligned} \tilde{\Pi}^{\text{OPE}}(p, p') &= -\frac{1}{12m_b \pi} \langle \frac{\alpha_s G^2}{\pi} \rangle \int d^4x d^4y e^{ip'y} e^{-ipx} \\ &\quad \times \text{Tr} [\gamma_5 S_c^{ja}(y-x) S_c^{ai}(x-y) \gamma_5 S_u^{ij}(-y)]. \end{aligned} \quad (73)$$

In other words, the correlator $\tilde{\Pi}^{\text{OPE}}(p, p')$ is suppressed by the factor $\langle \alpha_s G^2/\pi \rangle$. In what follows, we denote the corresponding invariant amplitude as $\tilde{\Pi}^{\text{OPE}}(p^2, p'^2, q^2)$.

In calculation of $\tilde{\Pi}^{\text{OPE}}(p, p')$, we set $m_u = 0$. The perturbative terms in all propagators lead to a contribution which is proportional to $\langle \alpha_s G^2/\pi \rangle$. A dimension-7 term in $\tilde{\Pi}^{\text{OPE}}(p, p')$ arises from the component $\sim \langle \bar{u}u \rangle$ in $S_u(x)$ and perturbative ones in $S_b(x)$. A contribution $\sim \langle \alpha_s G^2/\pi \rangle^2$ appearing due to $g_s G_{ab}^{\alpha\beta}$ components in b propagators and perturbative term in $S_u(x)$ is small and can be neglected. Higher dimensional terms in the quark propagators lead to effects suppressed by additional factors. It is seen that the strong coupling G_1 depends on the gluon vacuum condensate $\langle \alpha_s G^2/\pi \rangle$, which was fixed from consideration of the different hadronic processes [36,37,47,48]. In our study we use its value from Eq. (16) extracted in Refs. [36,37].

Operations required to extract SR for the strong form factor using correlator $\tilde{\Pi}^{\text{OPE}}(p, p')$ do not differ from ones explained in this article. Therefore, omitting details, we present final results for the decay $T_{\text{bc}\bar{\text{b}}\bar{\text{c}}} \rightarrow D^0 \bar{D}^0$. Numerical computations of the strong form factor $G_1(q^2)$ are carried out using the following Borel and continuum subtraction parameters in the D^0 channel: $M_2^2 \in [1.5, 3] \text{ GeV}^2$, $s_0' \in [5, 5.2] \text{ GeV}^2$. The coupling G_1 is extracted at the mass shell $q^2 = m_{D^0}^2$ and is equal to

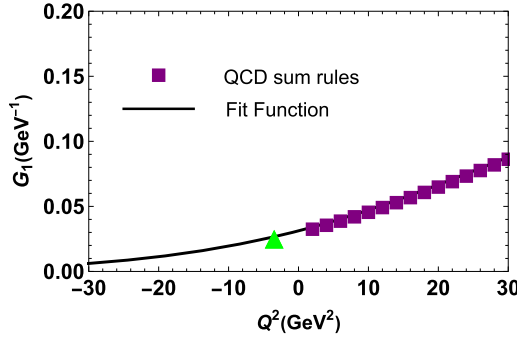


Fig. 4. Results of SR calculations and fit function for the form factor $G_1(Q^2)$. The triangle shows the point $Q^2 = -m_{D^0}^2$.

$$G_1 \equiv \tilde{G}_1(-m_{D^0}^2) = (2.68 \pm 0.42) \times 10^{-2} \text{ GeV}^{-1}. \quad (74)$$

To find G_1 , we have employed the SR data calculated in the interval $Q^2 = 2 - 30 \text{ GeV}^2$ and the fit function with parameters $\tilde{G}_1^0 = 0.031 \text{ GeV}^{-1}$, $\tilde{a}_1^1 = 7.58$, and $\tilde{a}_1^2 = -9.61$. The SR data and extrapolating function $\tilde{G}_1(Q^2)$ are shown in Fig. 4.

The width of this process can be obtained by means of the expression

$$\Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^0 \bar{D}^0] = G_1^2 \frac{m_{D^0}^2 \tilde{\lambda}_1}{8\pi} \left(1 + \frac{\tilde{\lambda}_1^2}{m_{D^0}^2}\right), \quad (75)$$

where $\tilde{\lambda}_1 = \lambda(m, m_{D^0}^2, m_{D^0}^2)$. Finally, we get

$$\Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^0 \bar{D}^0] = (7.6 \pm 2.1) \text{ MeV}. \quad (76)$$

Parameters of the decay $T_{bc\bar{b}\bar{c}} \rightarrow D^+ D^-$ should be very close to ones of the channel $T_{bc\bar{b}\bar{c}} \rightarrow D^0 \bar{D}^0$ a difference being connected with small mass gap between the mesons D^0 and $D^\pm$. We neglect such unessential difference and set $\Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^+ D^-] \approx \Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^0 \bar{D}^0]$.

4.2. $T_{bc\bar{b}\bar{c}} \rightarrow D^{*0} \bar{D}^{*0}$ and $D^{*+} D^{*-}$

Decays to vector charmed mesons are considered within the same scheme. Here, we start form analysis of the correlation function

$$\begin{aligned} \tilde{\Pi}_{\mu\nu}(p, p') &= i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T} \{ J_\mu^{D^{*0}}(y) \\ &\quad \times J_\nu^{\bar{D}^{*0}}(0) J^\dagger(x) \} | 0 \rangle, \end{aligned} \quad (77)$$

where the interpolating currents of the D^{*0} and $\bar{D}^{*0}$ mesons are defined by means of the formulas

$$\begin{aligned} J_\mu^{D^{*0}}(x) &= \bar{u}_i(x) \gamma_\mu c_i(x), \\ J_\nu^{\bar{D}^{*0}}(x) &= \bar{c}_j(x) \gamma_\nu u_j(x). \end{aligned} \quad (78)$$

The matrix elements which are necessary to calculate the correlator in terms of masses and decay constants of the D^* mesons are

$$\begin{aligned} \langle 0 | J_\mu^{D^{*0}} | D^{*0}(p') \rangle &= f_{D^*} m_{D^{*0}} \varepsilon_\mu(p') \\ \langle 0 | J_\nu^{\bar{D}^{*0}} | \bar{D}^{*0}(q) \rangle &= f_{D^*} m_{D^{*0}} \varepsilon_\nu(q), \end{aligned} \quad (79)$$

where $m_{D^{*0}}$ and f_{D^*} are the mass and decay constant of D^{*0} and $\bar{D}^{*0}$. The polarization vectors of these particles are $\varepsilon_\mu(p')$ and $\varepsilon_\nu(q)$. We introduce the vertex by the following way

$$\begin{aligned} \langle D^{*0}(p') | \bar{D}^{*0}(q) | T_{bc\bar{b}\bar{c}}(p) \rangle &= G_2(q^2) [q \cdot p' \\ &\quad \times \varepsilon^*(p') \cdot \varepsilon^*(q) - q \cdot \varepsilon^*(p') p' \cdot \varepsilon^*(q)]. \end{aligned} \quad (80)$$

Here, $G_2(q^2)$ is the form factor which correspond to the vertex $T_{bc\bar{b}\bar{c}} D^{*0} \bar{D}^{*0}$ and at the mass shell $q^2 = m_{D^{*0}}^2$ is equal to the strong coupling G_2 .

As a result, the $\tilde{\Pi}_{\mu\nu}^{\text{Phys}}(p, p')$ takes the form

$$\begin{aligned} \tilde{\Pi}_{\mu\nu}^{\text{Phys}}(p, p') &= \frac{G_2(q^2) \Lambda f_{D^*}^2 m_{D^{*0}}^2}{(p^2 - m^2) (p'^2 - m_{D^{*0}}^2) (q^2 - m_{D^{*0}}^2)} \\ &\quad \times \left[\frac{1}{2} (m^2 - m_{D^{*0}}^2 - q^2) g_{\mu\nu} - q_\mu p'_\nu \right] + \dots \end{aligned} \quad (81)$$

The correlation function $\tilde{\Pi}_{\mu\nu}(p, p')$ in terms of the quark propagators is equal to

$$\begin{aligned} \tilde{\Pi}_{\mu\nu}^{\text{OPE}}(p, p') &= i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle \bar{b}b \rangle \\ &\quad \times \text{Tr} \left[\gamma_\mu S_c^{ja}(y-x) \tilde{S}_c^{aj}(x) \gamma_\nu S_u^{ij}(-y) \right]. \end{aligned} \quad (82)$$

We use the invariant amplitudes which correspond to structures $g_{\mu\nu}$ in functions $\tilde{\Pi}_{\mu\nu}^{\text{Phys}}(p, p')$ and $\tilde{\Pi}_{\mu\nu}^{\text{OPE}}(p, p')$. In numerical analysis the Borel and continuum subtraction parameters in the D^{*0} meson channel are chosen equal to $M_2^2 \in [2, 3] \text{ GeV}^2$, $s'_0 \in [5.7, 5.8] \text{ GeV}^2$. The coupling G_2 is

$$G_2 \equiv \tilde{G}_2(-m_{D^{*0}}^2) = (1.93 \pm 0.33) \times 10^{-2} \text{ GeV}^{-1}. \quad (83)$$

Then, the partial width of the channel $T_{bc\bar{b}\bar{c}} \rightarrow D^{*0} \bar{D}^{*0}$ amounts to

$$\Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^{*0} \bar{D}^{*0}] = (7.2 \pm 1.9) \text{ MeV}. \quad (84)$$

It is clear that an approximate equality $\Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^{*+} D^{*-}] \approx \Gamma[T_{bc\bar{b}\bar{c}} \rightarrow D^{*0} \bar{D}^{*0}]$ is correct for these two decay modes as well.

Information collected in this and previous sections about partial widths of different processes allows us to evaluate the full width of the scalar tetraquark $bc\bar{b}\bar{c}$. For this parameter, we get

$$\Gamma[T_{bc\bar{b}\bar{c}}] = (142.4 \pm 16.9) \text{ MeV}, \quad (85)$$

which classifies $T_{bc\bar{b}\bar{c}}$ as a wide state.

5. Discussion and concluding notes

In this paper, we have presented results of our comprehensive analysis of the scalar four-quark meson with quark content $bc\bar{b}\bar{c}$. We have considered it as a diquark-antidiquark state made of scalar diquark and antidiquark components $b^T C \gamma_5 c$ and $\bar{b} \gamma_5 C \bar{c}^T$. We have calculated the mass and width of this structure and found them equal to $m = (12697 \pm 90) \text{ MeV}$ and $\Gamma[T_{bc\bar{b}\bar{c}}] = (142.4 \pm 16.9) \text{ MeV}$, respectively. Our studies have been carried out using the QCD SR methods. Thus, the mass and current coupling of $T_{bc\bar{b}\bar{c}}$ have been evaluated using the two-point SR approach.

Partial widths of different decay channels of the tetraquark $T_{bc\bar{b}\bar{c}}$ have been computed by applying technique of three-point SR method. This is necessary to estimate strong couplings at relevant tetraquark-meson-meson vertices. We have analyzed two types of decay processes. First, we have considered decays to $\eta_b \eta_c$ and $B_c^{(*)} B_c^{(*)}$ pairs which are fall-apart processes, where all constituent quarks are distributed between final-state mesons. Second type of decays are ones which become possible due to annihilation of $\bar{b}b$ to light quarks followed by creation of a pair of $D^{(*)} D^{(*)}$ mesons with appropriate charges and quantum numbers.

The masses of the tetraquarks $bc\bar{b}\bar{c}$ with different quantum numbers J^{PC} were evaluated in various publications [10,22,25,30–35]. In fact, in the relativistic quark model the mass of the scalar particle $bc\bar{b}\bar{c}$ was estimated 12813 MeV [10]. The predictions of Ref. [22], where the authors used the color-magnetic interaction, change in the range of

13396 – 13634 MeV. A potential model which includes the linear and Coulomb potentials, and spin-spin interactions leads to the mass spectrum of the scalar particle varying within limits 12854 – 13024 MeV [25]. In Ref. [34] the authors applied the QCD SR method and found that the mass of the diquark-antidiquark state with $J^{PC} = 0^{++}$ is equal to 12.28 – 12.46 GeV depending on an interpolating current used in computations. The lowest prediction 10.72 GeV for the mass of the scalar state $bc\bar{b}\bar{c}$ was made recently in Ref. [35].

Our result overshoots considerably the SR predictions made in Ref. [34], being close to one from [10] and to lower limits found in some other publications. One of main reasons to explore the structures $bc\bar{b}\bar{c}$ is intention to find a state stable against strong decays. Therefore, in many articles authors compare obtained results for the mass with various two-meson thresholds. But, as we have demonstrated in previous section, without regard to output of such analysis, four-quark mesons $bc\bar{b}\bar{c}$ are strong-interaction unstable particles. Our estimates show that width of $T_{bc\bar{b}\bar{c}}^-$ generated by annihilation subprocesses $\bar{b}b \rightarrow \bar{q}q$ is not small and form at least 20% of its full width.

Further detailed investigations of the tetraquarks $bc\bar{b}\bar{c}$ are necessary to calculate their parameters and find reactions where they may be observed.

Declaration of competing interest

We declare that there is no conflict of interests regarding this manuscript.

Data availability

No data was used for the research described in the article.

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