



Exemplification process in online education: a longitudinal study of mathematics teachers

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Abstract

The examples used in the teaching–learning process of mathematics have a crucial role in fostering conceptual understanding, and some variables can affect instructors’ qualified example usage. This longterm study focused on mathematics teachers’ exemplification process in face-to-face and online learning environments. In this regard, the change in examples used by mathematics teachers were evaluated in terms of content preparation and presentation during the shift from face-to-face lectures to online classes. A longitudinal design was used in the study and the teaching processes of 14 middle-school mathematics teachers were observed over two semesters. Observation notes, course documents, and semi-structured interview data were analyzed, and content analysis findings were presented through descriptive statistics in order to compare content preferences in two different learning environments. The use of worked examples decreased, while the use of conceptual examples increased with the shift from face-to-face lectures to online classes. Also the length of time devoted to examples in online classes decreased, and examples were more teacher-centered. The interview revealed that mathematics teachers need support in terms of example preparation and presentation aspects in online learning environments. The other technological-pedagogical competencies that teachers might need to choose qualified examples in different teaching–learning environments are discussed in the light of relevant literature.

Keywords Exemplification · Mathematics teachers · Online education · Teaching practices

Introduction

One of the indicators explaining the effectiveness of the learning–teaching process is the quality of the interaction between the teacher, the student and the knowledge components. The pedagogical content knowledge of teachers, interactions with students, and presentation tools of knowledge in the different learning environments (e.g. face-to-face, hybrid, or online) can affect teaching practices (Borba et al., 2018; Francis & Jacobsen, 2013; Trenholm et al., 2016). Since the day Covid-19 started, many countries decided to switch from traditional to online education for different classroom levels (from K–12 to postgraduate

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level). After the outbreak of the pandemic, more emphasis has been put on how online learning environments can be used more effectively in the transition process between textbook, teacher and student knowledge (Demir et al., 2021; Rapanta et al., 2020; Sevimli et al., 2022). Because the discipline of mathematics has an abstract nature and accumulated knowledge structure, as well as requiring problem-solving processes, the content preferences of teachers in the Online Learning Environments (herein after OLE) can differ according to the Face-to-Face Learning Environments (herein after FLE).

Examples are one of the most important teaching contents of mathematics, because they have the potential to combine the abstract nature of mathematics and the problem-solving process with the visualization, proof and argumentation process in different learning environments (Mason & Pim, 1984; Rø & Arnesen, 2020; Watson & Chick, 2011). In this sense, determining which types of examples are used by teachers in the online education process and addressing the change in the time allocated to the examples will enable examination of the technological-pedagogical preferences of instructors. A growing body of literature has examined the role of examples (student-generated or teacher-centered) in mathematical understanding in face-to-face education (Bentley & Stylianides, 2017; Peled & Zaslavsky, 1997; Zaslavsky, 2019). Until recently, there was no answer for the question about how mathematics teachers choose mathematical examples and present that content in changing learning environments. Whereas it is well-known that the type of examples chosen in the teaching can differ depending on the type of knowledge to be taught, the teacher's experience, and the teaching environment (Atkinson et al., 2000; Rowland, 2008; Sevimli, 2016). Also, the advantages offered or the limitations caused by the changing learning environments can affect the presentation methods of the examples, which are the important content of mathematics, in the teaching process. Especially after the Covid-19 pandemic, there is the need for research in this area to determine the time allocated to examples and how mathematics teachers' choices of examples have changed in online classes.

The main purpose of this research was to examine how the examples used by middle-school mathematics teachers have changed in terms of content preparation and presentation during the Covid-19 pandemic. The research questions were formulated as follows: (1) how did the presentation methods of examples used by mathematics teachers change during the shift from face-to-face lectures to online classes? and (2) how did the types of examples used by mathematics teachers change during the shift from face-to-face lectures to online classes? By answering these research questions, knowledge about the characteristics of the examples prepared by mathematics teachers for use in classroom practices would be obtained. Besides, determining the time allocated for examples in online learning environments compared with face-to-face classes would provide an opportunity to understand teachers' presentation methods of the examples. In this way, the quality of the teaching content used by teachers during the exemplification process could be evaluated from the perspective of the advantages and limitations of the OLE.

Conceptual framework

Exemplification in mathematics education

In a dictionary entry, an example is a noun that means “a little piece taken from the whole to illustrate the characteristics of it”. Examples are used to describe a concept or

phenomenon, as well as to show the nature of a general rule (Mason & Pim, 1984). In educational literature, a glossary of example terms includes teaching content, both in the context of a textbook and in the context of teacher narration and student explanations. The key role that examples play in the process of developing and teaching mathematics as a discipline has required greater emphasis on the exemplification process of research on mathematics education compared with other fields (Watson & Mason, 2005). In parallel with researchers' increasing interest in this issue, there have also been different variations in the definition of the concept. For instance, the mathematics education literature includes a special representation chosen to explore or explain a general principle (Watson & Chick, 2011), a tool used to explain mathematical knowledge (Bills et al., 2006), a kind of practice technique that allows conceptual communication between the student and the teacher (Peled & Zaslavsky, 1997), and content that allows the general to be seen in the particular (Mason & Pimm, 1984). Upon synthesizing the characteristics that all these definitions reveal for examples, it is observed that examples can have different roles such as summarization, explanation, interpretation, and confirmation.

Several classifications for examples have been developed to understand the characteristics of examples in mathematics education. In early stages, examples were divided into two categories such as generic (generalizable examples aimed at clarifying the subject) and specific (limited examples used to explain more specific cases) (Mason & Pim, 1984). With a similar perspective, Watson and Mason (2005) pointed to contents that emphasize the critical features of a concept while defining the boundary examples. The classification that is most used in the mathematics education research is that formed by Bills et al. (2006) who divide examples into generic examples, counter-examples, and non-examples. Counter examples can be used to counter a hypothesis or assertion, while non-examples can serve to clarify the boundaries of a concept. Rowland (2008) considered the use of examples by teachers under the categories of variables, sequencing, representations and learning objective, suggesting that novice teachers need specific guidance and assistance in appreciating the different roles of examples in mathematics in teaching. Across these categories of examples, there are also three other special labels of example types that researchers independently observe during the teaching process: prototype example, start-up example, proof example (Peled & Zaslavsky, 1997; Sevimli, 2016; Watson & Mason, 2005; Zaslavsky, 2019). While studies of the use of examples as a proof or argumentation tool have increased in the last decade (Rø & Arnesen, 2020), the classification was built on the use of examples during generalizing and disproving processes (Yopp & Ely, 2016).

The presentation method of the examples also can be an indicator factor for categorization. For instance, an example can be present as the teacher-centered methods (teacher is dominant in the process of sharing content) or learner-generated (students are actively involved in the process of generating, solving, or interpreting examples) methods. The related literature has demonstrated that it is important for learners to generate and solve examples and also for the types of examples presented in the classroom (Bentley & Stylianides, 2017; Watson & Chick, 2011). Much of the current literature on the exemplification process in mathematics education pays particular attention to the following topics: (1) the determination of the characteristics of the examples used in the teaching–learning (Bills et al., 2006; Sinclair et al., 2011; Watson & Chick, 2011), and (2) evaluation of the role of exemplification processes on mathematical reasoning or proving (Bentley & Stylianides, 2017; Peled & Zaslavsky, 1997). However, there are no previous reports in the literature on what kind of changes have occurred in the choice of examples in technology-supported teaching environments or how teachers' choice of examples is shaped in the process of reflecting on knowledge to be taught (textbook knowledge) to knowledge actually taught

(teacher practice). Examples are crucial reference sources for comprehending the knowledge provided by teachers in the classroom and for knowing which sources are used for what purpose in the process of didactic transposition of scholarly knowledge (Chevallard & Bosch, 2013). In the next section, the didactic characteristics of these environments are discussed in order to evaluate reflections of the exemplification process in online learning environments.

Teaching mathematics in online learning environments

With the frequent use of Internet technology and interactive digital tools in the teaching process, classes have started to involve virtual, online, or digital adjectives next to their names. These terms actually mean to conduct education on web-based platforms. With the shift from traditional (physical) classes to virtual classes, new pedagogical information began to be needed. Because it is not only enough to integrate digital tools into the classroom in the OLE, but it is also necessary for teachers to be able to prepare suitable teaching content and present this content in accordance with the constructivist approach in these classes (Engelbrecht et al., 2020). Digital pedagogy stands out as a skill that teachers should have for planning, implementing, and evaluating teaching in the OLE, especially in online or blended learning approaches. Digital pedagogy is a domain of competence based on a constructivist approach aimed at effective use of modern technologies in the teaching–learning environment. Digital pedagogy includes dimensions such as digital literacy, system management skills, knowledge of appropriate digital teaching tools and creating effective teaching content, while the present study focuses specifically on the dimension of preparing effective teaching content. To ensure effective learning environments, teachers need to prefer and use more pragmatically digital resources when teaching content in live online classes (Bennison et al., 2020). Criteria that can be considered to assess whether effective teaching content is presented in accordance with appropriate pedagogical approaches when teaching mathematics are whether pedagogically-high cognitive content is used, and how it is presented and scheduled (Trenholm & Peschke, 2020).

A series of recent studies have indicated that mathematics teaching practices in the OLE differ from the traditional classroom (face-to-face lectures) and that technological-pedagogical competencies shape applications in mathematics classes (Arcavi, 2020; Bennison et al., 2020; Borba et al., 2018). With the help of tablets with flexible writing tools that allow online sharing (Karal et al., 2015), and with the support of web 2.0 tools designed particularly for mathematical learning areas (such as dynamic geometry software and online data analysis platforms), mathematical concepts, symbols and solution process steps can now be easily presented to students interactively in the OLE. Arcavi (2020) also pointed out that digital tools used in mathematics education increase the capabilities of teachers to access and manage teaching content. Moreover, Moodley (2019) claims that effective use of online social media networks can support the professional development of teachers in virtual classrooms. Apart from advantages, the difficulties encountered during the mathematics teaching process in the OLE also are discussed in the literature. For instance, when Demir et al. (2021) examined the attitudes and opinions of the teachers in mathematics lessons conducted with online education during the first year of Covid-19 pandemic, they determined that teachers had difficulties, especially with technical problems and lack of interaction. It was also reported in the literature that interaction is limited when teaching mathematics in the OLE, communication tools are insufficient and gestures/facial expressions cannot be observed (Trenholm & Peschke, 2020). Mathematics, which

has more symbolic language, can cause communication limitations because of the lack of software or hardware in the process of teaching online classes. In this context, instructors cannot use drawing tools flexibly when presenting content (text adherence or limited typing flexibility with the Mouse), and the learning management system (LMS) does not have a discussion web-log or cooperative learning activities. When the experiences of instructors in the compulsory transition to online education in undergraduate level mathematics were examined, Sevimli (2022) found that content that requires the proof of integral theorems and the use of integration techniques in virtual classrooms was reduced by lecturers because of the limitations of menu tools and more dependence on the calculus textbook in the OLE.

Upon analyzing the relevant literature, no previous research was found to have evaluated the process of preparing effective teaching content, which is one of the technological-pedagogical competencies, through online classroom practices of teachers. After the Covid-19 pandemic, the impact of the transition from the FLE to the OLE on teacher practice is still important. During this urgent and unprepared transition, it is unclear whether teachers have changed their teaching practices in the FLE and prepared content appropriate to the OLE, and how teachers have prepared these contents. This study was conducted to determine the types of examples used in the teaching of mathematics in the online education process, and thus to understand the technological-pedagogical competencies needed for effective teaching of content.

Method

Research design and participants

The examples preferred by mathematics teachers in two different learning environments were examined in this study, with a longitudinal study being used in the design of the method. The longitudinal study is a pattern of developmental research that requires working with the same participants over an extended period of time to reveal the changes in their specific behavior in different environments. Participants in the study were mathematics teachers and their choices of examples were obtained via a longitudinal design by accessing the same group of individuals (minimizing individual differences). Because the teachers' previous experiences and the type of schools in which they work also could affect the exemplification process with the learning environments, the study participants were selected according to purposeful sampling. The participants comprised 14 mathematics teachers working in five different middle schools in the Middle Black Sea Region of Turkey. Nine participants were female and five were male, and all participants had professional experience between 11 and 20 years. All participants declared that they participated in a course (2 h) with web 2.0 tools within the scope of inservice training before the pandemic.

Factors affecting school choice are socioeconomic status and cooperation protocol. Before participant teachers were selected, a pool was created and preliminary interviews were held with the Psychological Counseling and Guidance Services in these schools. Within the scope of the interview, the data that can be accepted as an indicator for socioeconomic levels of the school students were collected. According to this, the students in the schools where the participants work are similar according to the following characteristics: the rate of having a personal computer (62–70%), the rate of having an internet connection (80–93%); and the average years of education of the members living in the

family (8–10 years). The student profile in the schools included in the sample represents the national average in terms of family income. The use of a smart board in the FLE, and access to the internet in the OLE, can affect teacher preparation and student motivation. For this reason, the schools where participants work are located in the city center, and these schools use a smart board during face-to-face education. Within the scope of the cooperation protocol, preservice teachers were given the opportunity to practise in the relevant school and the academics made observations at the school.

As part of the study, the examples used by mathematics teachers in two different learning environments were examined and the fall semesters of the 2019–2020 and 2020–2021 academic years were included in the research. In the spring of 2020, the transition from face-to-face training to online education occurred because of the Covid-19 pandemic but, in the first place, training was carried out with television broadcasts instead of the OLE in Turkey. The spring semester was not considered in the research process in order to prevent data loss in the process of comparing learning environments and also because of the lack of sufficient motivation and time devoted for teachers to prepare content. In addition, it was possible to study again with the same participants during two semesters (fall semesters of the 2019–2020 and 2020–2021 academic years), by virtue of the fact that there is a cooperation protocol between the middle schools. Presenting the characteristics of learning environments contributes to a better understanding of the findings obtained within the scope of the research. For this reason, general features related to face-to-face lectures and online classrooms are included in the next section.

Settings

In this study, we presented a general picture of the experiences of mathematics teachers regarding content preference during the sudden transition to the OLE from the FLE. For this reason, it is important to understand how the change in the learning environment reflects on the content preference of participating teachers under the conditions that the professional competencies of the teachers and the socioeconomic levels of the students have similar characteristics. These characteristics were obtained through the researchers' observation notes and video recordings of the teaching process. The common characteristics of both the FLE and OLE can be listed as follows. Observation data supporting these characteristics are also given, respectively: (1) direct instruction method (FLE: 47%; OLE: 51%) and question–answer technique (FLE: 39%; OLE: 45%) are often implemented more by the participants; (2) the order of the units of study and the length of time devoted to the unit are similar in both learning environments (with the time devoted to natural numbers, sets and fractions being common in the curriculum and the participants adhering to it throughout the teaching period); (3) the introduction of units or topics begins with activities, and then examples and definitions are presented (In all units in the 6th grade mathematics textbook, the introduction of the concept begins with an activity); and (4) middle school mathematics textbooks printed by the Ministry of Education were sent to students free of charge.

Characteristics of learning environments in which research data were collected were also examined according to their differences. The characteristics that distinguish FLE and OLE can be listed as follows: (1) classroom attendance was higher in the FLE than in the OLE (According to the course attendance chart at FLE, the average attendance rate was 94%. The live courses attendance rate was 80% in the statistics of OLE); (2) participating teachers chose supplementary sources in addition to the textbook in the FLE (all

participants) while, in the OLE, 11 out of 14 participants used web-based educational portals in addition to individual narration; (3) compared to the FLE, students were given more assignments in the OLE; and (4) students' generated examples decreased during the transition from FLE (29%) to OLE (12%) and there was an increase in the teacher-centered example presentations.

One participant in the OLE shared a document camera, while five participants shared a graphic tablet, and eight participants shared pdf annotator tools (write notes, edit drawings, solve problems, etc.) as teaching content. Teachers accessed online classes through the e-content platform, for which the Turkish acronym is EBA (The Educational and Informatics Network). EBA includes live video conference applications with supportive teaching materials such as video, games, z-books, TV, radio, discussion platforms and also teachers can be streamed live course on online cloud-based platforms such as ZOOM Cloud Meeting (Fig. 1). Within the context of this study, live lecture content in the OLE and teaching processes in the classroom in the FLE were considered.

In order to understand the research framework, it is important to understand the level of the content presented in teaching environments and the description of the learning area studied. Middle-school education lasts for 4 years (Grade 5–8) in Turkey, and there are five mathematical learning areas at this level. The study was carried out through the 'numbers and operations' learning area of 6th-grade mathematics. Grade 6 was selected for the study to reduce the effect of the familiarization phase of students on the teaching content preference during the transition from the classroom teacher to the branch teacher (from primary to the middle-school level). The content presented in Table 1 illustrates the average length of follow-up time of the lectures observed in the learning area of numbers and operations. While determining the distribution of lecture hours followed in learning environments, the time allocated to the sub-learning areas in the annual plans was taken into account. These

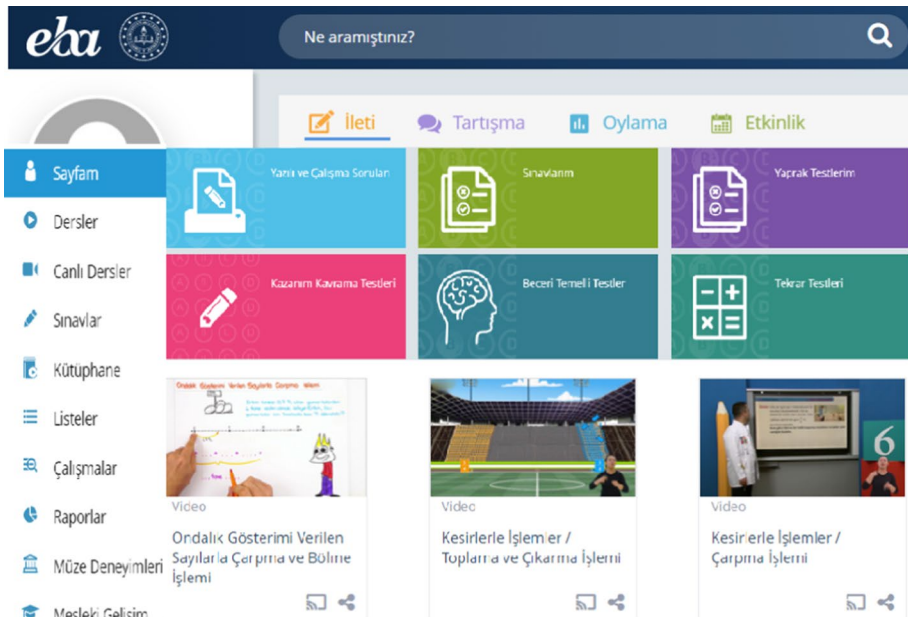


Fig. 1 An image from the EBA platform

Table 1 Distribution of lecture hours in learning environments

Learning area	Sub-learning area	Lecture hours	
		FLE	OLE
Numbers and operations	Operations with natural numbers	7	6
	Factors and multiples	8	11
	Sets	5	5
	Integers	5	4
	Operations with fractions	8	9
Total		33	35

data show that lectures given in both FLE and OLE are significantly represented and that a similar number of lectures have been selected in such a way as to ensure the validity of the content. In the scope of the study, an average of 33 lecture hours were provided to FLE participants and an average of 35 lecture hours were provided in the OLE, so that a total of 462 lectures were observed in the FLE and 490 lectures were observed in the OLE.

Data-collection process

In the data-collection process, it is necessary to determine the types of examples that middle-school mathematics teachers prefer to use in their lectures and to determine the issues affecting the choice of examples. From this viewpoint, observing multiple teachers only at a specific lecture time, or observing a limited number of teachers for a large number of lectures, could limit the trustworthiness and transferability of the study. Observing the lessons of different teachers in different schools over the lecture of a teaching period is quite time-consuming and a process that few researchers can handle alone. It was decided to collect data over on the grounds that it would be incomplete to conduct analysis through the participants' lecture plan or lecture notes (the teacher does not reflect some content in the lecture notes, uses technology support, etc.). The data in this research were obtained over multiple observation sources for the periods of two years and within the scope of the School Experience course, which is given in the last year of teacher training programs in Turkey and includes observations and activities made by preservice mathematics teachers over a period of 12 weeks under the guidance of a master teacher. The theoretical part of this course (for one hour each week) was conducted by the researcher and the master teachers were observed during the practical part of the course (for four hours each week). This course has an ideal design for the research data collection strategy because it allows a large number of live lecture observations and interviews with teachers. In this study, 16 preservice mathematics teachers enrolled in the School Experience course participated in the workshop on 'characteristics and taxonomy of examples used in mathematics teaching' in the theoretical part of the lecture, and the examples used in the mathematics textbook were classified together with the preservice mathematics teachers to enable them to gain experience. Preservice mathematics teachers who had received necessary training to evaluate the different example types used in the teaching of mathematics observed teachers participating in the practical part of the School Experience course, as well as accompanying the researcher in the data-collection process of this research. Thus, the examples used

by participating teachers in classroom practices were obtained as a result of synthesizing observation reports of multiple preservice mathematics teachers.

Data-collection tools

In the present study, the exemplification processes of mathematics teachers in teaching practices were evaluated under the themes of content presentation and preparation processes. For this purpose, data related to the length of time devoted to the use of examples and types of examples used were achieved through observations and interview techniques.

Lesson observation form

The Lesson Observation Form (LOF) was needed to access standardized data in lectures, observed by researchers and preservice mathematics teachers. The LOF was designed to include observation notes for two criteria (types of examples used and duration), which were considered in this study. The exemplification framework in the mathematics education literature, field observations, and expert opinions were considered when developing the LOF. First, because it is necessary to determine the types of examples used by teachers in the teaching process, the classification of examples by Bills et al. (2006) and Smith and Stein's (1998) taxonomy were blended together, building a classification that also refers to the epistemological characteristics of examples as well as their pedagogical qualities. This classification schema synthesizes the purpose of using examples by teachers in classroom practice and the reflection of scholarly knowledge in the textbook through examples. According to this classification schema, the four main types of examples are start-up, worked, conceptual, and proof examples. The length of time devoted to the use of examples in the class is one of the criteria, which is recorded in the LOF. In field observations, it was determined that the time spent for presentation and solution of examples was at least 1–9 min. In that sense, the “duration” criterion was also added to the LOF to determine the average time devoted to each example using an equal interval classification criterion of two minutes.

With the usage of LOF, the teaching content of each participant was coded for 12 weeks (three hours for each week). In addition, the researcher directly followed the lessons of the participating teachers along with the preservice mathematics teachers for some weeks, with the validity and reliability of the observations being confirmed again by comparing them with the student notebooks. In this way, the vast majority of the examples in the dataset included the content shared by multiple sources (the researcher's note along with the preservice mathematics teachers' observations and the students' notebooks). In the selection of the student notebook, the students who took the most-comprehensive notes and largely matched the observers' previous observation notes were included in the data of the study. A total of 90 examples in textbooks, 2436 used in the FLE and 3290 used in the OLE, were evaluated within the scope of the study.

Semi-structured interviews

Semi-structured interviews were carried out after the LOF data were collected and used to support these data. The interview form was used for all participants to understand their views on the advantages and limitations of the OLE during exemplification process. During the preparation of the interview form, the findings of the LOF were used in the context

of duration and types of used examples. In this sense, the experiences of the participants in the process of preparing and presenting the examples used in online learning environments were questioned. The interviews were conducted through an online conference program and recorded with permission. In the interview protocol, it was stated that the participants could leave the interview at any time, some examples of the content that they used during the OLE were presented, and their consent was obtained by sharing the personalized LOF findings. In the interview, which lasted between 20 and 30 min for each interviewee, participants were asked what are the advantages and limitations of the OLE in (1) the presentation of mathematical examples; and (2) the preparation of mathematical examples? After the interview questions were prepared, the opinion of two experts who completed their PhD in mathematics education was obtained. The suggestions given by experts about the participants' reactions to their the LOF findings were taken into consideration. Interview transcripts were also sent to the participants and, after their approval, the analysis phase was started.

Data analysis

Two analysis methods were used for two types of data (observation notes and interviews). The categorical data obtained by the LOF in classroom observations were analyzed through descriptive statistics and presented via percentage-frequency distributions. The LOF, which was the primary data-collection tool of the research, was used by the researcher and 16 pre-service mathematics teachers synchronously to observe the lessons of participating teachers. As it can be seen in Table 2, each example used by the teacher in the classroom was first numbered before coding was carried out according to the criteria of sub-learning area, types of example used, duration, and sources used. Types of examples used by participating teachers in both environments (FLE and OLE) were encoded under one of the example types (start-up, worked, conceptual, or proof) and represented by descriptive statistics. Start-up examples were employed to clarify a concept and present the characteristics of the concept. For instance, an example presented to show that the value of the distance will not be negative when introducing the concept of absolute value can have a start-up type. Worked examples are used to practise and calculate to reinforce the mathematical concept

Table 2 The example classification criteria in the LOF

Sub-learning area: Operations with numbers

Example

The population of Turkey about 80 000 000 in 2018. We also know that there were about 28 personal cars per 100 people in that year. How many personal cars were there in Turkey in 2018?

Solution

We should know how many group of 100 there are in 80.000.000.

$$80.000.000 \div 100 = 80.000$$

After that, Let's multiply each group with 28.

$$80.000 \times 28 = 2.240.000$$

Types of example

Start-up	Worked	Conceptual	Proof
	✓		

Duration

$0 < t \leq 2$	$2 < t \leq 4$	$4 < t \leq 6$	$t > 6$
	✓		

Presentation method

Teacher-centered	Student-centered
✓	

Sources

In textbook	Out textbook
	✓

being taught. For instance, after the concept of absolute value is given, the calculation of absolute values of integers between -5 and 0 can be evaluated within the context of the worked example. Conceptual examples contribute to establishing a relationship between different knowledge and concepts. For instance, calculating the total length of the path taken by two vehicles moving away from each other in the opposite direction and at different speeds, with a starting point of 0 , and proportioning the paths taken connects the concepts of absolute value and fraction. Proof examples are used to generate arguments to convince other people that something is true, and they include tasks such as generalization, inference, and inverse exemplification. It can be intuitively felt by students with proof examples that the sum of the absolute values of any two integers must be less than the absolute value of the sums of these numbers (triangle inequality). This classification schema, which categorizes the examples that teachers use in classroom practice, is important because it both takes previous classifications as a reference and is structured according to the teachers' reasons for the choice.

The presentation methods of examples as the teacher-centered (teacher is dominant in the process of sharing content) or student-centered (students are actively involved in the process of generating, solving, or interpreting examples) were also shared with a percentage rate. The place of the examples in the teaching contents was also examined over the duration (in minutes) allocated to each example. While analyzing the interview data, the responses of each participant to the interview questions were coded and the distribution of the participants' responses in the framework of the common categories corresponding to these codes were described via the table. The opinions upon which at least two participants agreed were taken as a reference in categorization processes. The interview data gathered from the participants were evaluated under the advantages and limitations categories in line with the questions in the interview form. Under these categories, participants' views of content preparation and presentation processes in online classes were coded together with sub-categories. Because some participants had views about both advantages and limitations, participants were coded under more than one category. The examples in the figures were translated from Turkish to English by the researcher to describe the teaching process in the learning environments.

Validity and reliability

In this study that was structured according to the qualitative approach, credibility, transferability, and consistency processes were evaluated respectively for validity and reliability. The method used for the credibility of the research is as follows. First, the longitudinal research design was used in the study and the data collection-analysis process took two years, enabling us to establish a long-term interaction with the data. Second, multiple observations were carried out with the same participants through purposeful sampling to ensure that the data could be transferred to other situations. The physical characteristics of learning environments and the participants' teaching experiences in these environments were described in detail via observation (under the Setting heading), thus determining the transferability framework of the findings. During the observation process, because the lectures considered the introduction to units and/or concepts, the study focused on the ways of using examples regarding concepts used in the teaching process. In this way, although the lectures followed through observation notes correspond to a third of the semester, the fact that the observed lectures were introductory to the concept/unit increased the generalizability of findings. In addition, even though a limited number of teachers (14 mathematics

teachers) took part in the study as participants, it was thought that comprehensive findings would be achieved in the selection of these teachers because of the requirement set by the Ministry of National Education (in Turkey) to be master teachers.

In order to ensure the consistency of the research, methods of diversification in the data source, multiple observation notes in the data-collection process, and inter-rater reliability in the data analysis process were used. Therefore, student notebooks and textbooks were considered along with observations when selecting the examples participants used in the classroom practice. In addition, the diversification of data sources was ensured by checking the consistency of observation findings with the interview process. The researcher's biases were reduced by considering the notes of more than one observer in the lecture follow-up. The coding consistency of different evaluators, who were trained in the classification of example types and processed observation notes into standardized forms, were checked. The reliability coefficient was calculated by the Miles and Huberman's (1994) inter-rater reliability formula $\left[\frac{\text{number of subjects on which consensus is reached}}{\text{number of all subjects}} \times 100 \right]$. Consequently, 278 randomly-selected examples were encoded by three different raters and, as a result of cross-comparisons, the inter-rater reliability coefficient ranged from 0.81 to 0.88. These results indicate that the coding for characteristics of examples was highly compatible. In order to check whether there was any data loss in the observation notes of the prospective teachers, student notebooks in the FLE and lecture videos in the OLE were examined; both sources showed that observation notes had high comprehensiveness.

Findings

While presenting the descriptive findings, the examples used by participants in the FLE and OLE were primarily considered in terms of time devoted to example presentation. Later, the results of the example types used in different learning environments were evaluated by comparing them with the textbook. The interview findings were presented categorically to facilitate understanding of the advantages and limitations of the OLE compared with the FLE during the exemplification process.

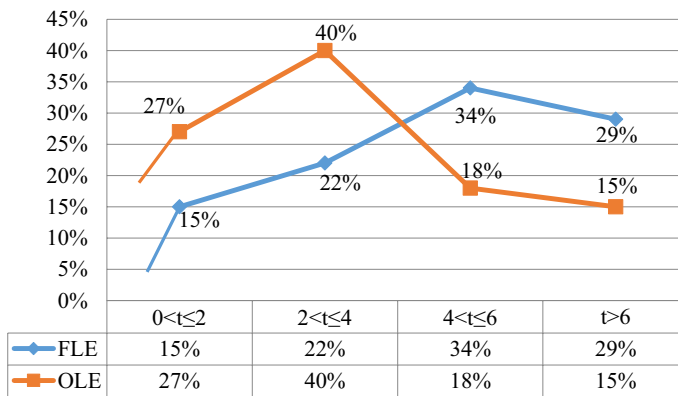
Duration and presentation methods of examples

One of the most-important objectives of the research was to evaluate the examples that mathematics teachers use when teaching during their lectures in the FLE and OLE. We determined the average number of examples used per lecture and the total number of examples in each learning environment. The findings related to the analyzed contents showed that the average of the total number of examples used by per participant was 174 in the FLE, whereas this number was 234 in the OLE (Table 3).

It was also found that the average number of examples used each lesson was 5.2 in the FLE and 6.7 in the OLE. This finding suggests that participants used up to a third more examples in the OLE than in face-to-face lectures. In order to scrutinize this finding more thoroughly, the time devoted to examples in teaching environments was also examined. The average time participants devoted to an example for presentation was 4.5 min in the FLE and 3.3 min in the OLE. The data available in Fig. 2 include the durations (in minutes) that participants used for presenting examples in the FLE and OLE. Participants devoted between 4 and 6 min to 34% of the examples that they used in the FLE. In the FLE, examples lasting longer than 6 min include 29% of all examples, while examples

Table 3 Frequency of characteristics of the analyzed contents

Characteristics	Frequency	
	FLE	OLE
Total observed lesson hours	462	490
Average observation hours for per participant	33	35
Total number of examples in lecture notes	2436	3290
Average number of examples used by per participant	174	234
Average number of examples used in per lesson	5.2	6.7
Average minute devoted to an example	4.5	3.3

**Fig. 2** Distribution of the time devoted to the examples

lasting less than 2 min constitute 10% of all examples. It was found that participants used examples requiring 2–4 min in the OLE in the teaching process (40%). Examples lasting less than 2 min are also often available in the OLE (27%). When comparing the two learning environments, it was observed that participants frequently use examples taking less than two minutes in the OLE, while they rarely tend to use examples taking longer than 6 min. Besides, it was found that the use of examples requiring more than 4 min in the OLE showed a trend of decreasing compared with the FLE.

The participants' teaching content preferences can be affected by the way in which the content is used in classroom practice, as well as the length of time devoted to the content. To determine the presentation method, the use of each example in the FLE and OLE is classified as teacher-centered or student-centered. When the two learning environments are compared in terms of presentation methods, student-centered presentations take more time in the FLE compared to the OLE (Fig. 3). In this sense, 21% examples were completed in 4–6 min and 17% of examples last 6–8 min under the student-centered presentations in the FLE. Also student-centered presentations usually were completed in a short span of time (between 2 and 4 min) in the OLE. When the time allocated to teacher-centered presentations were evaluated, it was found that the exemplification process was completed in a shorter time in the OLE, but no significant difference was encountered in terms of the time allocated to teacher-centered presentations in the FLE. In this regard, 22% of the examples were completed in between 2–4 min and 17% of examples lasted between 0–2 min under

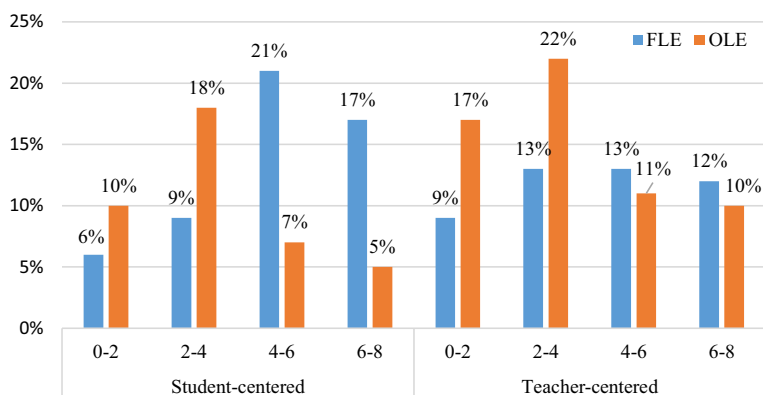


Fig. 3 Distribution of the duration and presentation method of examples

the teacher-centered presentations in the OLE. When the findings in Fig. 3 were evaluated as a whole in terms of the learning environment versus presentation methods, it can be seen that 53% of the examples were student-centered while 47% were teacher-centered in the FLE. In the OLE, teacher-centered examples were more frequent than student-centered examples.

To compare the exemplification process of the same participant in different learning environments in terms of the presentation-duration relationship, two examples that P_7 used while explaining the fraction topic were shared. For instance, the example of fraction in Fig. 4 used by P_7 in the FLE was first presented to students with a worksheet and then it took about 4 min with a student explaining his solution on the board. While the student solved the example on the board, the other students wrote in their notebooks.

The example used for the same topic (fractions) in the OLE was completed in less than 3 min, although it requires more steps and solutions (Fig. 5). P_7 used a learning toolkit to simulate the process of adding denominators in unequal fractions through the

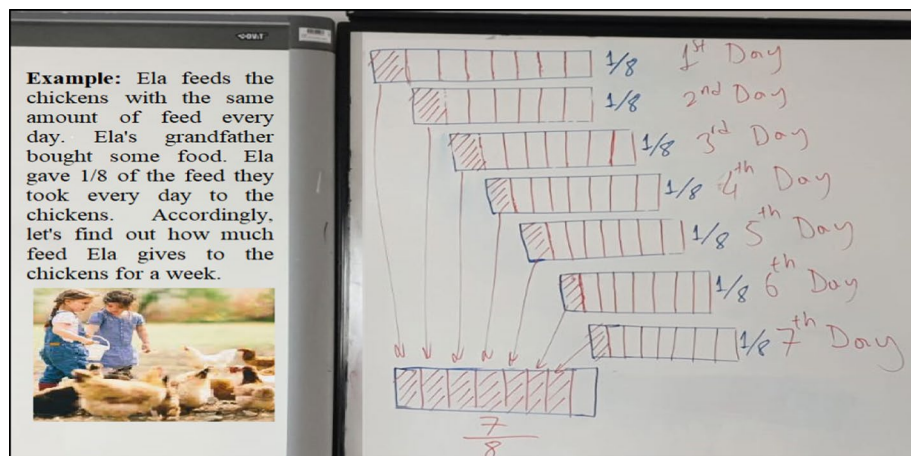


Fig. 4 A student-centered example used by P_7 in the FLE

Example: Erdem first gave six out of 10 marbles and then one fourth of his marbles to his friend Enes. What is the ratio of the marbles that Erdem gave to Enes to the marbles he had at the beginning?

Geogebra

Kesirleri Toplama

☐ Kesir Çıkarma ☐ Kesir Değiştir ☐ Genişlet ☒ Sol Sağ

$$\frac{6}{10} + \frac{1}{4}$$

Kesirleri Toplama

☐ Kesir Çıkarma ☐ Kesir Değiştir ☒ Genişlet ☒ Sol Sağ

$$\frac{2 \times 6}{2 \times 10} + \frac{1 \times 5}{4 \times 5} = \frac{12}{20} + \frac{5}{20}$$

Kesirleri Toplama

☐ Kesir Çıkarma ☐ Kesir Değiştir ☒ Genişlet ☒ Sol Sağ

$$\frac{2 \times 6}{2 \times 10} + \frac{1 \times 5}{4 \times 5} = \frac{12}{20} + \frac{5}{20} = \frac{12+5}{20} = \frac{17}{20}$$

Kesirleri Toplama

☐ Kesir Çıkarma ☐ Kesir Değiştir ☒ Genişlet ☒ Sol Sağ

$$\frac{2 \times 6}{2 \times 10} + \frac{1 \times 5}{4 \times 5} = \frac{12}{20} + \frac{5}{20} = \frac{12+5}{20} = \frac{17}{20}$$

Chat
Breakout Rooms
Invite
Record
Hide Video Panel
Share computer screen
Optimize Share for
End Meeting

Fig. 5 A student-centered example used by P₇ in the OLE

Geogebra software and she shared a link with the students for simultaneous participating in the solution of the example.

When participants were asked about the advantages and limitations of online environments in terms of the time devoted to the examples within the scope of the interview, the sub-categories in Table 4 were obtained. The sub-category to which the participants referred more frequently under the advantages of online teaching environments category was the availability of Synchronized learning tools (6 out of 14 participants). For instance, P₉ stated that the availability of web 2.0 tools in online environments offers the opportunity for students to work together (see Table 4). In addition, four participants considered as an advantage that Recorded lectures can be presented to students without wasting time, referring to the fact that the content can be re-watched according to students' own learning speed in the OLE. For instance, P₁₄ considers the opportunity to return to the recorded content as an advantage in terms of using more examples and being able to share video content in and out of class. Consequently, when the participants' views of the advantages and limitations of the OLE were compared in terms of content presentation, it was determined that the participants referred more to the technological advantages offered by online classes (Table 4).

When participants were also asked why they prefer more teacher-centered presentations, although student-centered presentations require less time in the OLE, answers generally focused on the limitations of OLE. Under the limitations category, the sub-categories of classroom management and student motivation were identified. Four participants (P₃, P₇, P₁₀, and P₁₂) pointed out that they preferred teacher-centered presentations to student-centered ones because of difficulties in classroom management. Two participants, on the other hand, stated that they use examples that do not require a long time to be presented in the OLE, citing the decrease in motivation over time. For instance, P₁ pointed out that student motivation is low over the long term in the online course and therefore he shared the worksheets which contained examples after live online classes.

Table 4 Participant views on the advantages and limitations of the OLE in terms of content presentation

Categories	Sub-categories	Participants	Excerpt from participants
Advantages	Synchronized learning tools	P ₂ , P ₇ , P ₅ , P ₆ , P ₉ , and P ₁₃	P ₉ : I think it is an advantage that math-based applications such as Geogebra or Mathigon, where students can be engaged simultaneously on the same example, are easily available in online classes
	Recorded lectures	P ₄ , P ₁₁ , P ₁₂ , and P ₁₄	P ₁₄ : It can take time to wait for the examples to be written by the students. Since the lessons are recorded in live online classes, the students can return to the course content they want, and I was able to use the increased time by giving more examples in or out of class
Limitations	Classroom management	P ₃ , P ₇ , P ₁₀ , and P ₁₂	P ₁₀ : I had to give some content, which I solved with the collaborative approach in face-to-face classes, with direct instruction in online education. Because it is difficult to provide classroom management in online classes and to determine who participates in the example solution process in detail
	Motivation	P ₁ and P ₈	P ₁ : It is not easy to keep students' attention in front of the computer screen for a long time. When I asked students a question, very few are motivated to answer. I shared most of examples in the worksheets after the course, so students can take a deeper look at the teaching content

Types of examples used

Another purpose of this research was to examine the types of examples used by teachers in different learning environments. Upon examining the descriptive results of the distribution of examples in Table 5, the most-common type of example involved in the 6th-grade mathematics textbook, which is commonly used by participants, is the worked example (58%). It was found that 28% of examples in the textbook were conceptual examples, but that a limited number of start-up and proof examples were available. Upon examining the examples chosen by the participants in the teaching process in the FLE, it was observed that about two out of every three examples used in the classroom were worked-type examples, followed by conceptual examples. While more than half of the examples chosen by participants in the OLE were still of the worked type, the proof example was rarely used in these classes.

Comparing the learning environments, it was found that the types of examples which participants chose in both teaching–learning processes were similar to each other, but there were differences between the frequency of choices of examples according to the learning environment. Even though the most-commonly chosen example type in both the FLE and OLE was the worked type, there was a decrease in the percentage of preferences of worked types, while there was an increase in the choice of conceptual types, when switching from face-to-face lectures to online classes. While the utilization rate of the start-up example in the OLE was higher than for the FLE, the least-chosen type of example in both learning environments was the proof example. When the examples that are used in learning environments are compared to the textbook, the most obvious change in the FLE is the decrease in the conceptual type of examples whereas there is a remarkable change in the OLE in terms of the start-up type of examples. To examine the reflection of the examples that are contained in textbooks on the teaching practice, how many of these examples were used in the FLE and in the OLE also was investigated. Textbook examples were used more often in the OLE when compared with the FLE. It was determined that 47% of the examples in the textbook were used in the FLE while 76% of examples were used in the OLE by direct quotation.

It was also ascertained through interviews how online learning environments affected the participants' content (example) preparation processes. Five participants stated that online learning environments contributed to the process of preparing examples in terms of resources diversity (Table 6). Participants, who were evaluated their opinions in this sub-category, pointed out that qualified examples can be presented to students without wasting time via e-textbooks that can be easily integrated into the OLE. For instance, P3 stated that video-supported examples in the OLE were chosen from the EBA platform so that a wide

Table 5 Example types used in learning environments

Learning environment	Example types								Agreement rate of example	
	Start-up		Worked		Conceptual		Proof			
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Textbook	10	11	52	58	25	28	3	3		
FLE	26	15	113	65	30	17	5	3	42	47
OLE	51	22	119	51	61	26	3	1	68	76

* *n*: the average number of examples used per participant

Table 6 Participant views on the advantages and limitations of the OLE in terms of content preparation

Categories	Sub-categories	Codes/participants*	Excerpt from participants
Advantages	Resources diversity	P ₂ , P ₃ , P ₇ , P ₁₂ , and P ₁₃	P ₃ : I was able to easily share different types of examples on digital platforms or ready-made electronic resources with the students during the online lesson. For instance, I was able to support the topics in the "numbers and operations" learning area with available examples, which were supported with video on the EBA platform
	Personalized content	P ₅ , P ₈ , and P ₁₁	P ₁₁ : I give students the opportunity to generate their own examples in the breakout rooms. Thus, we were able to generate personalized contents and share them in the classroom in a short time
Limitations	Content-oriented platforms	P ₁ , P ₃ , P ₄ , P ₆ , P ₉ , P ₁₀ and P ₁₄	P ₆ : Online learning platforms or virtual manipulatives generally include start-up examples. It can be difficult to find content suitable for each outcome in the curriculum. For this reason, I would like to use the content that has been developed specifically for the learning outcomes
	Synchronized writing tools	P ₂ , P ₅ and P ₁₂	P ₂ : There are some examples in my lecture notes that I used as prototypes in the lessons, but I could not use them in online classes because we suddenly switched to online education. Since the writing tools only allowed me to write, I could not solve the worked type examples with the participation of students

variety of examples could be delivered to students faster. When asked about the effect of the shift from face-to-face lectures to online classes on the chosen sources during access to examples, P2 and P7 stated that they benefited from the online platforms (e.g. CODAP, Learning apps, Pool Everywhere, etc.) in order to use different type of representations in the OLE during the exemplification process. P5, P8 and P11 considered it as an advantage that more-personalized content could be generated in the OLE through interactive group work. Thus, the teacher can organize suitable content for different levels of students, and students can choose appropriate examples from among these content in the OLE. When the participants' views on the advantages and limitations of the OLE were compared in terms of content preparation, participants referred more to the limitations caused by the changing learning environments (Table 6; Fig. 6).

Under the limitations category, the sub-categories of content-oriented platforms and synchronized writing tools were identified. Half of the participants stated that content-oriented platforms that are compatible with the curriculum are limited or that they are not aware of the qualified resources available, if any. Three participants, on the other hand, stated that they could not use the examples in their lecture notes because of the lack of synchronized writing tools. For instance, P₆ stated that she referred to web sources that can be used more flexibly in the OLE, but that she does not have clear knowledge of their compatibility with the curriculum because she has reached these contents via her own experience. P₂, on the other hand, stated that he benefited from his personal example space in the FLE, while he stays more committed to the textbook in the OLE. Figure 5 includes two different types of examples about fractions that P₂ uses in face-to-face lectures and online classes. It seems that P₂ reflects a proof-type example in a face-to-face classroom using the white-board, but presents a start-up example in an online classroom via web 2.0 tools. When it was asked why he (P₂) gave limited places in both classes to proof examples, P₂ stated that he carried out the introduction to the 'numbers and operations' in the FLE via tasks and thus used examples after the definition of the concept by practising instead of providing knowledge about the concept. According to him, it is more difficult to present tasks in the OLE. For this reason, P₂ stated that there is more need for the use of start-up examples that include the definition of the concept in the OLE.

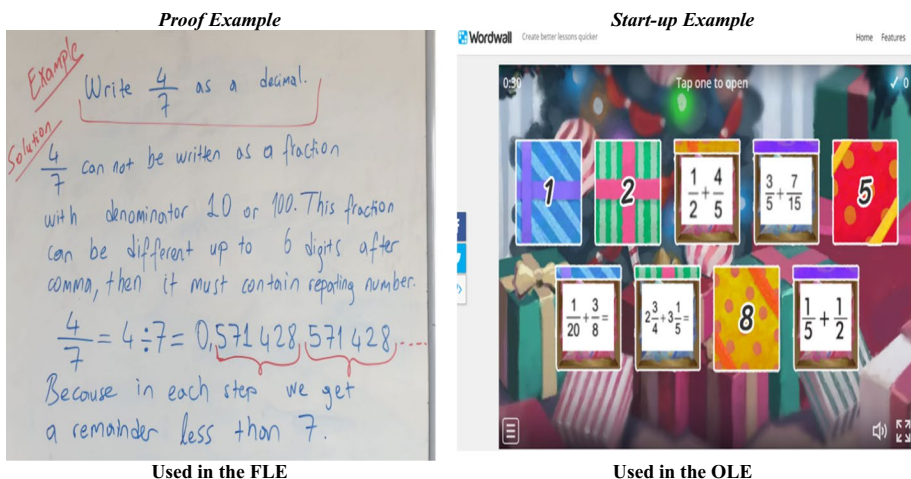


Fig. 6 Example types used in the classrooms

Discussion

The research findings were discussed under the themes of content presentation and preparation processes. While the findings regarding the presentation methods of the examples were discussed, the time allocated to the examples and the number of used examples were emphasized. The main findings regarding the presentation of the examples showed that more examples were used in the OLE compared with the FLE, and that the average length of time devoted to the presentation of an example in the OLE decreased. Taking into account the average number of examples (5.2 for FLE and 6.7 for OLE) given by participants per lesson and the average time devoted to an example (about 4.5 min for FLE and 3.3 min for OLE) in the classroom practices by participants, it seems that example content is used in more than half of the lessons (lesson duration is 40 min) in both learning environments. Based on this finding, it can be concluded that examples are an important component of online mathematics teaching as well as face-to-face instruction (Bills et al., 2006). One of the reasons why the participants used more examples in the OLE might be that they needed more examples when presenting the mathematical definition or that they included less of other content (tasks, exercises, proofs, etc.) in the teaching process. The findings supporting this inference were actually encountered in the interviews. One of the participants (P_{10}) stated that, instead of introducing the concept with teaching contents (tasks, problem situations, etc.) that require collaborative work, lectures are organized through giving definitions and presenting examples. Nonetheless, in accordance with the constructivist approach, the student should be able to reach an understanding with their previous knowledge instead of presenting the definition of the concept directly. The teaching content in the online classrooms should be organized in collaborative participation to foster conceptual understanding and promote mathematical communication (Borba et al., 2018).

The finding that participants use more examples in the OLE compared with the FLE might provide the basis for two inferences. First, with the technological advantages offered by online classes, teaching content (examples) can be brought together with the students in a shorter period of time (Arcavi, 2020; Trenholm et al., 2016). In the interview findings, participants frequently stated that they benefited from the advantages of the OLE by using digital learning tools and recording the content. The second inference is that there is a change in the way in which the examples are presented. In this study, how examples are presented in classroom practice was also examined. The example presentation, which is student-centered and more suitable for the constructivist approach, decreased with the transition from face-to-face lectures to online classes and a significant rate of the examples were used in the dominance of teachers. From this perspective, in an environment where teacher–student interaction is more limited (OLE), teachers can be considered to have a more-dominant role in managing the teaching process. The greatest limitation of online classes compared with face-to-face classes is the lack of interaction (Borba et al., 2018; Demir et al., 2021; Trenholm & Peschke, 2020). In fact, it is possible for the student to take the floor and generate examples to carry out solutions on the virtual board in the OLE, which in short means that the lectures are interactive. To increase interaction, Borba et al. (2018) suggest that interactive digital teaching materials should be prepared in the OLE to foreground student-centered teaching in accordance with the constructivist understanding. From the beginning of the Covid-19 pandemic, teachers in Turkey accessed online classes through the e-content platform, which is known as EBA and is freely available. But it was observed that

participants do not benefit from the advantages offered by this platform. This finding also indicates that participants need support in choosing and using effective teaching content, which is one of the technological-pedagogical competencies.

The other theme that represents the focus of the discussion is the type of examples that teachers use when preparing the teaching content. Participating teachers used worked types of examples in the FLE, while they used conceptual and start-up examples more frequently in the OLE. It is more important to use different types of examples with similar rates in the class than to use a single type of examples (Sevimli, 2016; Watson & Chick, 2011). From this point of view, it can be claimed that the example diversity in the OLE is greater and that therefore more rich contents are preferred. The finding distinguishing this study from previous research is that, with the transition to online classes, the use of conceptual examples that support relational understanding increases, while the use of the worked type of examples decreases. Because of the preference for textbook-compatible content in the OLE, more varied types of examples might have been used in these classes. As a matter of fact, when the sources from which the examples were selected were examined, it was found out that the examples in the textbook were more directly used in the OLE. Similarly, Sevimli et al. (2022) found that the frequency of use of textbooks by middle-school mathematics teachers during the transition from face-to-face education to distance education changed in that teachers used the textbook more often in distance education. Another reason for more-diverse example types being used in the OLE is that digital sources can be used flexibly in these classes. Although teachers have been caught unprepared for the transition to online classes, materials supported by internet technology can be used in these classrooms, which contain various types of examples. Provided that these teaching materials are defined specifically to textbooks, it is important that the interactive-dynamic teaching contents that can be used in online environments are integrated with internet technology (video-supported, QR-coded, web 2.0-integrated contents).

Just as online education offers an advantage in terms of content preparation, inability to use teaching habits in online classes might also have affected teachers' content preferences. For instance, the decline in participants' digital writing ability, their lack of sufficient knowledge, and lack of time to prepare content in the OLE also might have forced them to adhere more to the textbook. Taking into account that participants in this research frequently used the pdf version of the textbook in the OLE, it can be confirmed that they more often refer to teacher-centered approaches because of their inability to use digital writing tools flexibly. The interview data also revealed that some participants had difficulties because of a lack of writing tools while sharing the examples of their previous lecture notes in online classes, and therefore criticized the students' not taking notes. The limited teacher–student interaction and lack of teachers' knowledge of choosing the appropriate digital tool can be an obstacle to the use of effective content teaching in the OLE. Similar to the findings of Francis and Jacobsen's (2013), participants in the current study requested professional development on web 2.0 tools that can be used in mathematics classes in order to reflect the various sources offered by online classes on effective teaching practice. This finding indicates that teachers are aware of their shortcomings in terms of the technological-pedagogical aspects and are motivated to participate in professional development seminars in order to improve the quality of teaching in the OLE. On the other hand, Li et al. (2022) found that teachers involved in the online inservice teacher training course felt less satisfied with their experience of motivation and interaction during the training. Before the Covid-19 pandemic, the mathematics teachers in this study also participated in an online inservice teacher training course about the use of web 2.0 tools. Based on the findings of the present research and the results of previous studies, it is better to engage

teachers with interactive digital learning tools that can enable with the mathematical examples, instead of explaining the general use of web 2.0 tools in inservice training.

Conclusion and suggestions

The study suggests that the length of time devoted to examples in the OLE was shorter than in the FLE, but also that examples were used more frequently in these classes. Besides, students-centered methods are often preferred in the FLE while teacher-centered methods are preferred in the OLE during the content presentations. Differentiation in the learning environment has also affected teachers' content preparation process. It was determined that worked examples were used frequently in the FLE, whereas start-up and conceptual examples were preferred more often in the OLE. Use of proof examples, which require advanced mathematical thinking processes, was limitedly in both learning environments. Compared with face-to-face classes, types of example preferred by teachers in online education showed more similarity with the textbook examples. Interviews revealed that teachers needed the knowledge to use content-oriented digital learning platforms to provide variety in mathematical example types in technological aspects. It has also been concluded that teachers should be supported in terms of online classroom management to organize student-centered presentation methods interactively in pedagogical aspect.

Based on the results, some recommendations can be formulated for both researchers and educational practitioners. It is recommended that the content in both the textbook and teaching practice be further equipped with example types that require advanced mathematical thinking. Through inservice training, mathematics teachers can be aware of technologies that they can use more effectively in the OLE, and teachers can be supported to develop contents to enable interaction in the OLE. But it is not enough to form a framework for inservice training based only on the results of this research. Because in this research, limited learning areas, subject content, and examples were studied to make a holistic analysis. With other research in this area, technological content knowledge that teachers need in the OLE and their pedagogical beliefs can be examined in more detail. If the time devoted to teaching content had been studied separately in this study, the way in which examples are presented and the types of examples could have been examined together (accompanied by cross matches in future research), new contributions to the process of developing qualified teaching content might be provided.

The present study had certain limitations in terms of achieving more generalizable results. The first limitation is that only 14 mathematics teachers participated in the study and their lectures the Numbers and operations learning area were evaluated in terms of exemplification processes. It is believed that future research that involves evaluating different learning areas (such as algebra, geometry, statistics) and different teaching contents (such as tasks, definition, and/or exercises) over a broader sampling is likely to reveal generalizable results. Another limitation is that the online education approach was carried out with a fully-online model during the Covid-19 pandemic. Future studies could compare the types of examples used in different online learning environments (e.g. hybrid, open, flipped-classroom, etc.) after the pandemic. In this way, the technological and pedagogical aspects that affect the content preferences of teachers in different learning environments could be investigated comparatively.

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Declarations

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